

# Capital Requirements for Government Bonds – Implications for Bank Behaviour and Financial Stability

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## Abstract

This paper analyses whether the introduction of capital requirements for bank government bond holdings increases financial stability by making the banking sector more resilient to sovereign debt crises. Using a theoretical model, we show that a sudden increase in sovereign default risk may lead to liquidity issues in the banking sector. Our model reveals that in combination with a central bank acting as a lender of last resort, capital requirements for government bonds increase the shock-absorbing capacity of the banking sector and thus the financial stability. The driving force is a regulation-induced change in bank investment behaviour.

*JEL classification:* G28, G21, G01.

*Keywords:* bank capital regulation, government bonds, sovereign risk, financial contagion, lender of last resort.

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# 1 Introduction

During the financial crisis of 2007 onwards, significant contagion effects between sovereigns and banks could be observed. Serious doubts about the solvency of some EU member states put pressure on the balance sheets of banks with large sovereign debt exposures. In turn, significant bank bailouts strained public finances. This reinforcing feedback loop led to substantial risks to financial and macroeconomic stability. Against this background, there is an ongoing debate about whether the abolishment of the preferential treatment of sovereign borrowers in banking regulation can mitigate possible contagion effects from sovereigns to banks. Jens Weidmann, the president of the Deutsche Bundesbank, for example, strongly advocates in favour of the abolishment:

“There is one field in regulation, however, where too little has been done so far - the treatment of sovereign exposures in banks’ balance sheets. A banking system can only truly be stable if the fate of banks does not hinge on the solvency of their national sovereigns. Thus, I have been advocating, for quite some time now, a phasing-out of the preferential treatment of sovereign borrowers over private debtors.” (Weidmann (2016))

Compared to other assets, sovereign debt is given privileged treatment in banking regulation with respect to capital and liquidity requirements as well as to large exposure regimes. This paper deals with the preferential government bond treatment in capital regulation. Although the default probability of some EU member states is significantly higher than zero, banks do not have to back the government bonds of these countries with equity, these bonds are assigned a zero-risk weight in bank capital regulation.<sup>1</sup> Banks have multi-billion euro exposures to sovereign debt, in particular banks in stressed euro area countries have more than doubled their sovereign debt exposures in recent years (see Figure 1). However, banks’ sovereign holdings can act as a contagion channel through which sovereign distress can severely affect the banking sector. Considering, for example, the Greek sovereign debt crisis of 2009 onwards, the distressed state of public finances triggered fragility in the banking sector. Against this background the aim of this paper is

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<sup>1</sup>See CRR (Capital Requirement Regulation) Article 114. The CRR and CRD (Capital Requirement Directive) IV implemented the Basel III accords in EU law.

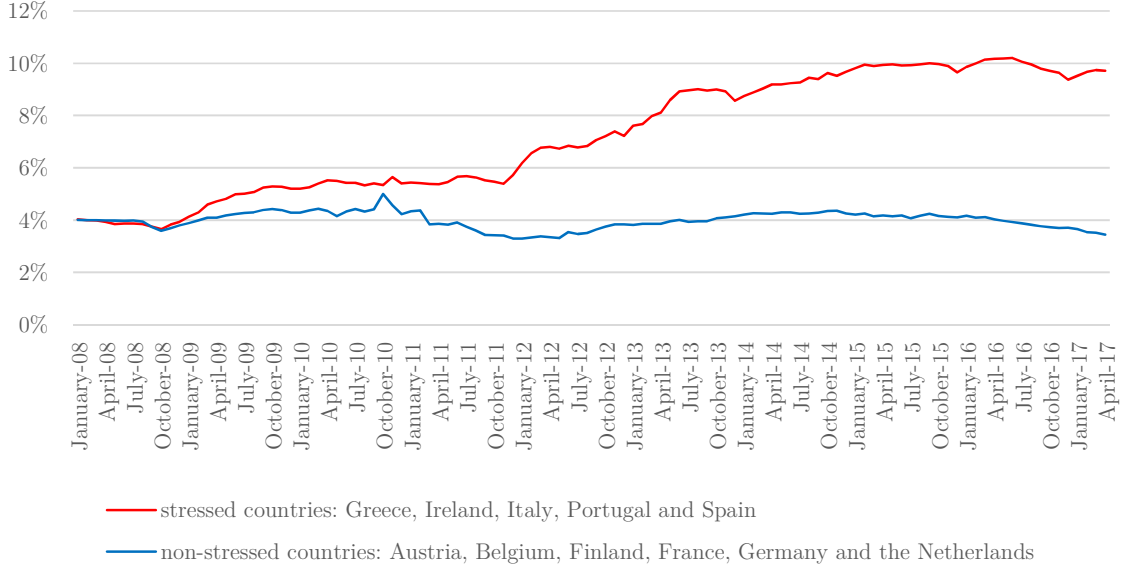


Figure 1: Banks' sovereign debt holdings as a percentage of their total assets in certain stressed and non-stressed euro area countries. *Data source: ECB.*

to investigate within a theoretical model whether the contagion channel from sovereigns to banks can be weakened through the introduction of capital requirements for government bonds, thereby making the banking sector more resilient to sovereign debt crises.

In a first step, we analyse the banks' investment and financing behaviour in different capital regulation scenarios. The banks' objective is to maximise their depositors' expected utility. The depositors have the usual Diamond-Dybvig preferences. In the banking sector, there is no aggregate liquidity risk, though banks do face idiosyncratic liquidity risks. Banks can invest in a risk-free short-term asset, which earns no return, and in two risky long-term assets (government bonds and loans) with an expected positive return. However, whereas loans are totally illiquid, government bonds are highly liquid as there exists an interbank market for this asset. Investing in government bonds thus allows banks to hedge their idiosyncratic liquidity risks.<sup>2</sup> Besides deposits, banks can raise equity capital to finance their investments. Raising costly equity capital allows banks to transfer liquidity risks associated with highly profitable but totally illiquid loans from risk-averse depositors to risk-neutral investors, thus increasing their depositors' expected utility. Within this model framework, it is shown that the introduction of capital requirements only for loans

<sup>2</sup>As pointed out by Gennaioli et al. (2014a), for example, banks may hold government bonds for many different reasons. So government bonds do play an important role in managing a banks' daily activities. In our model banks hold government bonds to manage their liquidity.

induces banks not only to raise more equity but also to increase their loan investment. The reason is that the regulation-induced reduction of the loan-to-equity ratio implies that the potential for a beneficial liquidity risk transfer from depositors to investors is no longer fully exploited. Banks mitigate the resulting negative effect on depositors' expected utility by increasing their investment in highly profitable loans in absolute terms. However, this means that banks must raise more equity to fulfil the capital requirements. Consequently, introducing capital requirements for loans implies that the loan-to-equity ratio decreases, but in absolute terms both loans and equity increase. This bank behaviour (increasing loan investment and raising more equity capital) will be reinforced if government bonds also have to be backed with equity. The crucial point is that this additional capital requirement implies that the loan-to-equity ratio will decrease even further so that the potential for a beneficial liquidity risk transfer can be exploited even less.

In a second step, we then investigate the banks' shock-absorbing capacity in different capital regulation scenarios. We suppose that the economy is hit by a shock in the form of an increase in the default probability of sovereign bonds (government bond shock). The increased doubts about sovereign solvency may lead to a sovereign bond price drop and hence to liquidity issues in the banking sector, leading to illiquid but per se solvent banks going bankrupt. We show that capital requirements themselves cannot prevent illiquid but solvent banks from going bankrupt. However, combined with a central bank as a lender of last resort (LOLR) that provides additional liquidity against adequate collateral, the introduction of capital requirements for loans and additional capital requirements for government bonds increase the banking sector's shock-absorbing capacity. The driving force is that the regulation-induced change in bank investment behaviour (more loans) implies that banks have more adequate collateral from which to obtain additional liquidity from the LOLR.

The rest of the paper is structured as follows. Section 2 presents the related literature. Section 3 describes the model setup. Section 4 analyses both sides of the interbank market for government bonds and derives the market equilibrium. Section 5 outlines the liquidity-risk-transfer property of equity capital and derives the banks' optimal investment and financing behaviour in different capital regulation scenarios. Building on these anal-

yses, Section 6 discusses the consequences of capital requirements for the shock-absorbing capacity of the banking sector and the importance of the central bank acting as a LOLR in this context. The final section summarises the paper.

## 2 Related Literature

Our paper contributes to three strands of literature. The first strand deals with financial contagion, the second with different institutions aiming to weaken the financial contagion channel between sovereigns and banks, and the third with the influence of capital requirements on bank behaviour.

In the literature, there is no single definition of financial contagion. We will refer to financial contagion if financial linkages imply that a shock, which initially affects only one or a few firms (financial or non-financial), one region or one sector of an economy, spreads to other firms, regions or sectors. In a seminal paper, Allen and Gale (2000) show that if there is an interbank deposit market which allows banks to balance their different liquidity needs, a small liquidity preference shock initially affecting only one bank may spread to other banks, leading to the breakdown of the whole banking sector. Allen and Carletti (2006) model contagion effects from the insurance to the banking sector. The crucial point is that the credit-risk transfer between these sectors implies that banks and insurance companies hold the same securities. A crisis in the insurance sector forces the insurance companies to sell these securities. The resulting price drop of these assets then also affects banks' balance sheets leading to severe problems in the banking sector. In a similar vein Heyde and Neyer (2010) show that credit-risk transfer within the banking sector may create a channel of financial contagion. Allen and Gale (2006) extend the Allen-Carletti model, enabling them to analyse the impact of bank capital regulation on systemic risk. They show that the introduction of binding capital requirements may increase systemic risk, as they induce inefficiencies for banks. These inefficiencies can be mitigated if banks share risks with the insurance sector. However, this risk-sharing increases contagion potentials between banks and insurance companies. Especially since the financial crisis of 2007 onwards, there has been a growing literature on financial contagion between sovereigns and

banks.<sup>3</sup> Gennaioli et al. (2014b) identify banks' government bond holdings as a potential link through which a sovereign default can severely affect the banking sector. However, they claim that it is exactly the existence of this potential contagion channel which makes the occurrence of a sovereign default less likely (and thus the occurrence of a banking crisis triggered by respective contagion). They argue that banks hold large amounts of domestic government bonds. This means that governments do not have an incentive to strategically default because a sovereign default would badly hit the domestic banking sector and thus the domestic economy.<sup>4</sup> In a similar way, Bolton and Jeanne (2011) analyse how much of an impact a sovereign default has on the banking sector in financially integrated economies. They find that, on the one hand, financial integration leads to risk diversification benefits for banks. However, on the other hand it generates a financial contagion channel between sovereigns. Acharya et al. (2014) investigate the two-way-feedback risk transmission between sovereigns and banks. They argue that government bank bailouts lead to a rise in sovereign credit risk. This in turn weakens the banking sector as the value of the banks' sovereign bond holdings and the value of their implicit and explicit government guarantees decrease. Cooper and Nikolov (2013) also examine the diabolic loop between sovereigns and banks and, in this context, the role played by fiscal guarantees and equity capital. As a policy implication the authors stress the role of equity capital as an important regulatory tool to isolate banks from sovereign risk and they suggest the implementation of capital requirements on sovereign exposures. Broner et al. (2014) argue that in turbulent times, sovereign debt offers a higher expected return to domestic than to foreign creditors. This creditor discrimination implies that banks increase their investment in domestic government bonds. If banks are financially constrained, which is especially the case in turbulent times, this bank investment behaviour will be in line with a decrease of private sector loans (crowding-out effect). This bank investment behaviour not only reduces economic growth but also reinforces the risk of financial contagion between sovereigns and domestic banks.

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<sup>3</sup>For a survey of the main channels through which sovereign risk influences the banking sector, see European Systemic Risk Board (2015).

<sup>4</sup>Acharya and Rajan (2013) analyse why countries have an incentive to serve their debt even if a government default would lead to little direct domestic cost. They argue that through a default, governments would lose access to debt markets, which would result in a decrease in fiscal spending and therefore in GDP, so that even short-horizon governments have an incentive to repay their debt.

There is also a rapidly growing empirical literature on financial contagion between banks and sovereigns. Acharya and Steffen (2015), for example, provide empirical evidence, that especially large, low-capitalised banks with risky assets tend to invest in long-term, risky, peripheral<sup>5</sup> government bonds, financing these investments by borrowing from the unsecured short-term wholesale market (“carry trade” behaviour). This financing and investment behaviour can be motivated by banks’ risk-shifting incentives and regulatory capital arbitrage as sovereign exposures are not subject to capital regulation. In a similar vein, Korte and Steffen (2015) empirically show that within the euro area, the zero-risk weight on (risky) sovereign debt exposures in banking regulation reinforces a potential financial contagion channel from foreign sovereigns to domestic banks, as the preferential treatment of government bonds in bank capital regulation subsidises risky sovereign debt holdings, so that banks hold a relatively large amount of these assets.<sup>6</sup> A huge number of empirical analyses confirms a strong link between the potential default of sovereigns and of banks by deriving a strong positive correlation between the CDS spreads of sovereigns and banks.<sup>7</sup>

Based on the literature on the sovereign-bank doom loop, the second strand of related literature discusses different newly implemented or proposed institutions aiming to weaken the financial contagion channel between sovereigns and banks. Covi and Eydarm (2016) argue that the *new recovery and resolution framework* actually weakens this contagion channel as, due to a “bail-in” rule, bank insolvencies no longer strain public finances. Farhi and Tirole (2016) claim that national governments may favour a lax banking supervision, particularly in times of a weak domestic banking sector, as losses in the banking industry can be shifted to international investors. They thus argue that a *shared supranational banking supervision* can diminish contagion effects between internationally operating banks and sovereigns. Brunnermeier et al. (2016) develop a model which illustrates how to isolate banks from sovereign risk via the introduction of *European Safe Bonds* (“ESBies”) issued

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<sup>5</sup>The term “peripheral” refers to the euro area (Greece, Italy, Ireland, Portugal, Spain, or GIIPS).

<sup>6</sup>Note in this context, that also Haubrich and Wachtel (1993) find empirical evidence that the introduction of a risk-adjusted capital ratio with a zero-risk weight on sovereign exposures makes sovereign bonds more attractive for banks in relation to loans. Accordingly, high-indebted banks in particular increase their sovereign bond investment at the expense of loans in order to meet the required capital ratio.

<sup>7</sup>See, for example, Acharya et al. (2014), Altavilla et al. (2017), Alter and Beyer (2014), Angelini et al. (2014), De Bruyckere et al. (2013), Ejsing and Lemke (2011), Gennaioli et al. (2014a), Gross and Kok (2013) or Vergote (2016).

by a European debt agency. They argue that holding these bonds disentangles banks from potential sovereign defaults, as ESBies are backed by a well-diversified portfolio of euro-area government bonds and are additionally senior on repayments.

The third strand of related literature deals with the influence of capital requirements on bank behaviour. Blum (1999), for example, points out that a binding required risk-weighted capital ratio may increase banks' risk taking behaviour.<sup>8</sup> Hyun and Rhee (2011) find that the introduction of a binding required risk-adjusted capital ratio may imply that banks reduce their loan supply (instead of increasing their equity capital) to fulfil the capital requirement. Harris et al. (2014) use a general equilibrium model to discuss the welfare consequences of higher bank capital requirements. They find that higher capital requirements may incentivise banks to invest in high-risk projects, inducing a decrease in overall welfare.

Our paper contributes to all three strands of literature: In our theoretical analysis, banks hold highly liquid government bonds to hedge their idiosyncratic liquidity risks. These government bond holdings generate a potential for financial contagion from sovereigns to banks (first strand). With respect to institutions aiming to weaken the financial contagion channel between banks and sovereigns (second strand), we derive that the introduction of binding capital requirements in general, and for government bond holdings in particular, are not sufficient to weaken this channel, but in addition, a central bank acting as a LOLR is necessary. Furthermore, we show how capital requirements influence bank investment and financing behaviour (third strand). The introduction of binding capital requirements only for loans obviously implies that the banks' loan-to-equity ratio decreases, but in absolute terms both loans and equity increase. This bank behaviour is reinforced if government bonds also become subject to capital requirements.

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<sup>8</sup>Flannery (1989) and Furlong and Keeley (1989) also analyse the risk-taking incentives associated with higher capital requirements. However, their analysis focuses on the role played by the existence of a deposit insurance.



### 3 Model

#### 3.1 Technology

We consider three dates,  $t = 0, 1, 2$  and a single all-purpose good that can be invested or consumed. At date 0, the all-purpose good can be invested in three types of assets: one short-term and two long-term assets. The short-term asset represents a simple storage technology i.e. one unit at date 0 returns one unit at date 1. The two long-term assets are government bonds and loans. However, unlike in other theoretical works, government bonds are not completely safe but yield a random return  $S$ . With probability  $1 - \beta$  the investment fails and one unit of the all-purpose good invested in government bonds at date 0 produces only  $l < 1$  units of this good at date 2. With probability  $\beta$ , the investment succeeds and produces  $h > 1$  units at date 2. A government bond is a liquid asset and can be traded at price  $p$  on an interbank market at date 1. The loan portfolio yields a random return  $K$ . If the loan investment succeeds, one unit invested at date 0 will generate a return of  $H > h > 1$  units at date 2 with probability  $\alpha < \beta$ . With probability  $(1 - \alpha)$  the investment fails and produces only  $L < l < 1$  units at date 2. The main characteristics of the loan portfolio are that it is the asset with the highest expected return as  $E(K) > E(S) > 1$ , the highest risk as the variance  $Var(K) > Var(S)$ , and that it is totally illiquid as loans cannot be traded at date 1. Banks discover whether the long-term assets succeed or fail at date 2. Table 1 summarises the returns on the different types of assets.

	Return at date 1	Return at date 2	
Short-term asset	1		
Government bonds	$p$	$\left. \begin{array}{cc} h & \beta \\ l & (1 - \beta) \end{array} \right\}$	$E(S) > 1$
Loan portfolio	0	$\left. \begin{array}{cc} H & \alpha \\ L & (1 - \alpha) \end{array} \right\}$	$E(K) > E(S), Var(K) > Var(S)$

Table 1: Return on the Different Types of Assets (Investment at Date 0: 1 Unit)

### 3.2 Agents and Preferences

In our model, there are three types of agents: a continuum of risk-averse consumers normalised to one, a large number of banks and a large number of risk-neutral investors. Each consumer is endowed with one unit of the all-purpose good at date 0. Like in Diamond and Dybvig (1983) consumers can be categorised into two groups. One group values consumption only at date 1 (early consumers), the other group only at date 2 (late consumers). We assume both groups are the same size. The proportion of early consumers is  $\gamma = 0.5$  and the proportion of late consumers is  $(1 - \gamma) = 0.5$ . Denoting a consumer's consumption by  $c$ , his utility of consumption is described by

$$U(c) = \ln(c). \quad (1)$$

However, at date 0 each consumer is unsure of their liquidity preference. He does not know whether he is an early or late consumer. Therefore, he concludes a deposit contract with a bank. According to this contract, he will deposit his one unit of the all-purpose good with the bank at date 0 and can withdraw  $c_1^*$  units of the all-purpose good at date 1 or  $c_2^*$  units of this good at date 2. As we have a competitive banking sector, each bank invests in the short-term asset and the two long-term assets in a way that maximises its depositors' expected utility.

While there is no aggregate liquidity risk (the fraction of early consumers is  $\gamma = 0.5$  for sure) banks are subject to idiosyncratic liquidity risk. Accordingly, they do not know their individual proportion of early consumers. With probability  $\omega$  a bank has a fraction  $\gamma_1$  of early consumers and with probability  $(1 - \omega)$  a bank faces a fraction  $\gamma_2$  ( $\gamma_2 > \gamma_1$ ) of early consumers, so that  $\gamma = 0.5 = \omega\gamma_1 + (1 - \omega)\gamma_2$ . As in Allen and Carletti (2006), we assume the extreme case in which  $\gamma_1 = 0$  and  $\gamma_2 = 1$ , so that  $\omega = 0.5$ .<sup>9</sup> Because of this strong assumption, we have a) two types of banks: banks with only early consumers (*early banks*) and banks with only late consumers (*late banks*) and b) the probability of becoming an early or a late bank is 0.5 each. Banks can hedge their idiosyncratic liquidity

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<sup>9</sup>The reason for this strong assumption is to keep the optimisation problem as simple as possible. Without this assumption the expected utility function given by (3) would be:  $E(U) = \omega\gamma_1\ln(c_1) + (1 - \omega)\gamma_2\ln(c_1) + \omega(1 - \gamma_1)[\alpha\beta\ln(c_{2Hh}) + \alpha(1 - \beta)\ln(c_{2Hi}) + (1 - \alpha)\beta\ln(c_{2Lh}) + (1 - \alpha)(1 - \beta)\ln(c_{2Li})] + (1 - \omega)(1 - \gamma_2)[\alpha\beta\ln(c_{2Hh}) + \alpha(1 - \beta)\ln(c_{2Hi}) + (1 - \alpha)\beta\ln(c_{2Lh}) + (1 - \alpha)(1 - \beta)\ln(c_{2Li})]$ . Given  $\gamma_1 = 0$  and  $\gamma_2 = 1$  the first and the last term of the equation can be eliminated.

risk by using an interbank market for government bonds: All banks invest in government bonds and the short-term asset at date 0. At date 1, when each bank has learnt whether it is an early or a late bank, it sells or buys government bonds in exchange for the short-term asset on the interbank market to balance its individual liquidity position.

In addition to the deposits, banks have the opportunity to raise funds (equity capital) from risk-neutral investors. These investors are endowed with an unbounded amount of capital  $W_0$  at date 0. The contract concluded between a bank and an investor defines the units of the all-purpose good (equity capital) which are provided at date 0 ( $e_0^* \geq 0$ ) and the units which are repaid (and consumed) by the investor at date 1 and date 2 ( $e_1^* \geq 0$  and  $e_2^* \geq 0$ ). As in Allen and Carletti (2006) the utility function of a risk-neutral investor is given by

$$U(e_0, e_1, e_2) = \rho(W_0 - e_0) + e_1 + e_2, \quad (2)$$

where the parameter  $\rho$  presents the investor's opportunity costs of investing in the banking sector.

### 3.3 Optimisation Problem

As ex-ante, i.e. at date 0, all banks are identical, we can consider a representative bank when analysing the banks' optimal investment and financing behaviour at date 0. Deposits are exogenous and equal to one. The bank has to decide on units  $x$  to be invested in the short-term asset, on units  $y$  to be invested in government bonds, on units  $u$  to be invested in loans and on units  $e_0$  to be raised from the risk-neutral investors. A bank's optimal behaviour requires the maximisation of the expected utility of its risk-averse depositors. Consequently, a bank's optimisation problem reads

$$\begin{aligned} \max E(U) = & 0.5\ln(c_1) + 0.5[\alpha\beta\ln(c_{2Hh}) + \alpha(1-\beta)\ln(c_{2Hl}) \\ & + (1-\alpha)\beta\ln(c_{2Lh}) + (1-\alpha)(1-\beta)\ln(c_{2Ll})] \end{aligned} \quad (3)$$

$$\text{with } c_1 = x + yp, \quad (4)$$

$$c_{2Hh} = uH + \left(\frac{x}{p} + y\right)h - e_{2Hh}, \quad (5)$$

$$c_{2Hl} = uH + \left(\frac{x}{p} + y\right)l - e_{2Hl}, \quad (6)$$

$$c_{2Lh} = uL + \left(\frac{x}{p} + y\right)h - e_{2Lh}, \quad (7)$$

$$c_{2Ll} = uL + \left(\frac{x}{p} + y\right)l - e_{2Ll}, \quad (8)$$

$$\begin{aligned} \text{s.t. } \rho e_0 &= 0.5(\alpha e_{2H} + (1 - \alpha)e_{2L}) + 0.5(\alpha\beta e_{2Hh} \\ &+ \alpha(1 - \beta)e_{2Hl} + (1 - \alpha)\beta e_{2Lh} + (1 - \alpha)(1 - \beta)e_{2Ll}), \end{aligned} \quad (9)$$

$$CR^{min} = \frac{e_0}{\phi_x x + \phi_y y + \phi_u u}, \quad (10)$$

$$e_0 + 1 = x + y + u, \quad (11)$$

$$x, y, u, e_0, e_{2Hh}, e_{2Hl}, e_{2Lh}, e_{2Ll} \geq 0. \quad (12)$$

Equation (3) describes the expected utility of the bank's depositors. With probability 0.5 the bank is an early bank, i.e. all of its depositors are early consumers who thus withdraw their deposits at date 1. In this case, the bank will use the proceeds of the short-term asset ( $x \cdot 1$ ) and of selling all its government bonds on the interbank market ( $y \cdot p$ ) to satisfy its depositors, as formally revealed by (4).

With probability 0.5, the bank is a late bank, i.e. all of its depositors are late consumers and withdraw their deposits at date 2. The consumption level of a late consumer depends on the returns on the bank's investments in government bonds and loans. As the probabilities of the success of these investments,  $\alpha$  and  $\beta$ , are independent, we can identify four possible states: both investments succeed, only the investment in the loan portfolio succeeds, only the investment in the government bonds succeeds, or both investments fail. We denote these four states simply as  $Hh$ ,  $Hl$ ,  $Lh$   $Ll$ . Equations (5) to (8) represent the consumption levels of late depositors in these possible states. The first term on the right-hand side in each of these equations shows the proceeds from the investment in loans, the second from the investment in government bonds. Note that the quantity of government bonds a late bank holds at date 2 consists of the units  $y$  it invested itself in government bonds at date 0, and of those it has bought on the interbank market in exchange for its units of the short-term asset  $x$  at date 1. The last term depicts the amount a bank has to

pay to the risk-neutral investors at date 2. Due to their risk-neutrality, they are indifferent between whether to consume at date 1 or at date 2. Consequently, optimal (risk-averse) deposit contracts require  $e_1^* = 0$ .

Equation (9) represents the investors' incentive-compatibility constraint. Investors are only willing to provide equity capital  $e_0$  to the banking sector if at least their opportunity costs  $\rho$  are covered. With probability 0.5 the bank is an early bank. Then, it will use its total amount of  $x$  including those units obtained in exchange for its total amount of government bonds on the interbank market to satisfy all its depositors at date 1, while investors will receive the total proceeds from loans  $e_{2H} = uH$  or  $e_{2L} = uL$  at date 2. With probability 0.5, the bank is a late bank. Then, the bank will buy government bonds on the interbank market in exchange for its short-term assets at date 1. At date 2, it will repay its depositors and investors. The investors will receive a residual payment from the proceeds of the bank's total loan and government bond investment, i.e. those returns not being used to satisfy the bank's depositors. Constraint (10) captures the capital requirements the bank may face. They are expressed as a minimum capital ratio  $CR^{min}$  of the bank's equity  $e_0$  to its (risk-)weighted assets  $\phi_x x + \phi_y y + \phi_u u$ . If  $\phi_x = \phi_y = \phi_u = 0$ , there will be no capital requirements. If  $\phi_x = \phi_y = 0$  and  $\phi_u > 0$ , there will be a privileged treatment of (risky) government bonds as only loans are subject to financial regulation. This privileged treatment will be repealed if  $\phi_y > 0$ . Then, risky government bonds will also have to be backed with equity capital. The budget constraint is represented in equation (11), and the last constraint (12) represents the non-negativity constraint.

## 4 Interbank Market for Government Bonds

Before solving the banks' optimisation problem in the next section, we will have a closer look at the interbank market for government bonds. Banks use government bonds to balance their idiosyncratic liquidity needs: At date 0 all banks invest in government bonds and at date 1 the early banks sell their government bonds to the late banks in exchange for the short-term asset. We assume that the late consumers' expected utility of

an investment in risky government bonds is higher than that of an investment in the safe short-term asset i.e.

$$\beta \ln(h) + (1 - \beta) \ln(l) \geq \ln(1) = 0. \quad (13)$$

This means that the expected return on government bonds is sufficiently higher than on the short-term asset to compensate the risk-averse depositors for the higher risk. If it were not for this assumption, an interbank market for government bonds would not exist as no bank would invest in government bonds.

At date 1, each bank has learnt whether it is an early bank or a late bank. However, late banks will only buy government bonds in exchange for their short-term asset if

$$\beta \ln(h) + (1 - \beta) \ln(l) - \ln(p) \geq \ln(1), \quad (14)$$

i.e. if the expected utility of their depositors is at least as high as that from the alternative of storing the short-term asset until date 2. Consequently, there is a maximum price

$$p^{max} = h^\beta l^{(1-\beta)} \quad (15)$$

late banks are willing to pay for a government bond. If  $p \leq p^{max}$ , a late bank wants to sell the total amount of its short-term asset in exchange for government bonds as government bonds yield a (weakly) higher expected utility for their depositors. If  $p > p^{max}$ , a late bank does not want to sell any unit of its short-term assets in exchange for government bonds.

Note that at date 0, all banks are identical and solve the same optimisation problem. Accordingly, for all banks the optimal quantities invested in the short-term asset and the long-term assets are identical. We denote these optimal quantities by  $x^*$ ,  $y^*$ , and  $u^*$ . Considering the number of depositors is normalised to one, the optimal quantities of each individual bank correspond to the respective aggregate quantities invested in each asset

type. As half of the banks are late banks, aggregate demand for government bonds at date 1 is

$$y^D = \begin{cases} 0.5 \frac{x^*}{p} & \text{if } p \leq p^{max}, \\ 0 & \text{if } p > p^{max}. \end{cases} \quad (16)$$

Figure 2 illustrates this demand function. The jump discontinuity at  $p^{max}$  results from the fact that for  $p \leq p^{max}$  late banks want to sell their total amount of the short-term asset  $0.5x^*$  in exchange for government bonds. The demand curve is downward sloping because the amount of liquidity in the banking sector which can be used for buying government bonds is limited to  $0.5x^*$ . Consequently, a higher price  $p$  implies that fewer government bonds can be bought. Independently of the price, early banks want to sell all their government bonds at date 1 as early consumers only value consumption at this time. Therefore, the aggregate supply of government bonds is perfectly price inelastic. The respective aggregate supply curve is given by

$$y^S = 0.5y^* \quad (17)$$

as illustrated graphically in Figure 2.

Considering (16) and (17) and denoting the equilibrium price for government bonds  $p^{**}$ ,<sup>10</sup> the market clearing condition becomes

$$\frac{x^*}{p^{**}} = y^*. \quad (18)$$

As there is no aggregate liquidity uncertainty and as all banks solve the same optimisation problem at date 0, aggregate supply and demand and thus the equilibrium variables are known at date 0. In addition, the following considerations reveal that  $p^{**} = 1$ . If  $p^{**} < 1$ , the return on government bonds at date 1 would be negative and thus smaller than on the short-term asset. Consequently, at date 0 banks would invest only in the short-term asset and not in government bonds. However, if no bank buys government bonds at date 0,

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<sup>10</sup>To be able to distinguish between those quantities optimally invested in the different assets and those quantities exchanged in equilibrium on the interbank market, we index optimal variables with  $*$  and interbank market equilibrium variables with  $**$ .

there will be no supply of government bonds and thus no interbank market for government bonds with a positive price at date 1.

If  $p^{**} > 1$ , a government bond would be worth more than the short-term asset at date 1. Therefore, no bank would invest in the short-term asset at date 0 but only in government bonds. However, if at date 0 no bank invests in the short-term asset but only in government bonds, there will be no demand for government bonds at date 1, and thus no interbank market for this asset with a positive price. Consequently, the only possible equilibrium price at date 1 is  $p^{**} = 1$ . Note that due to (13) and (15),  $p^{max} \geq 1$ , which implies that the interbank market is always cleared.

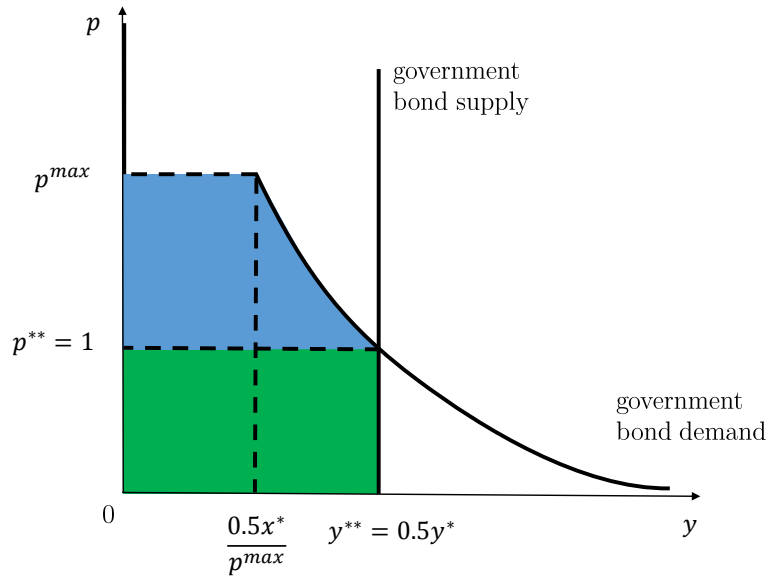


Figure 2: Interbank Market for Government Bonds at Date 1

Considering aggregate demand and supply curves allows us to determine the surplus of the banking sector from interbank trading. The equilibrium government bond trading volume is denoted by  $y^{**}$ . The blue area reflects the surplus of the late banks. They benefit from interbank trading as the exchange of the short-term asset for government bonds leads to a higher expected utility of their depositors (see equation (13)). The green area shows the surplus of the early banks from interbank trading. At date 1, government bonds produce no return so that their exchange in short-term assets allows for a higher date-1 consumption and thus a higher utility of early depositors.



## 5 Optimal Bank Investment and Financing Behaviour

This section analyses the impact of repealing the preferential treatment of government bonds in bank capital regulation on bank investment and financing behaviour. However, as a starting point we analyse bank behaviour without and then with the possibility of raising equity capital. This allows us to point to the key property of equity capital in our model, which is the property to transfer liquidity risk from risk-averse depositors to risk-neutral investors and thereby to increase depositors' expected utility. The subsequent analysis shows that the introduction of binding capital requirements for loans implies that the potential for liquidity risk transfer is no longer fully exploited which lowers depositors' expected utility. This effect will be reinforced if government bonds also have to be backed with equity. To compensate at least partially for this negative effect, banks invest more in highly profitable loans. To meet the required capital ratio, this changed investment behaviour works alongside raising more equity. Consequently, introducing capital requirements leads to a lower loan-to-equity ratio, but in absolute terms loan investment and equity capital increase.

To demonstrate a bank's optimal investment and financing behaviour in different regulation scenarios, we make use of a numerical example similar to the one used by Allen and Carletti (2006). The government bond returns  $h = 1.3$  with probability  $\beta = 0.98$  and  $l = 0.3$  with probability  $(1 - \beta) = 0.02$ . Consequently, the investment in government bonds of one unit of the consumption good at date 0 yields the expected return  $E(S) = 1.2746$  at date 2. Loans are also state-dependent and return at date 2. They return  $H = 1.54$  with probability  $\alpha = 0.93$ , and they fail and yield  $L = 0.25$  with probability  $(1 - \alpha) = 0.07$ . Hence, the expected loan return at date 2 is  $E(K) = 1.4497$ . Investors' opportunity costs are  $\rho = 1.5$ .

### 5.1 No Equity Capital

In the case with no equity capital, the constraints (9) and (10) are omitted, all  $e_{(\cdot)} = 0$ , and the budget restriction (11) becomes  $x + y + u = 1$ . Optimal banking behaviour in this case is shown in Table 2.

Balance Sheet

A			L		
$x^*$	0.5	50%	$D$	1	100%
$y^*$	0.5	50%			
$\Sigma$	1	100%	$\Sigma$	1	100%

Deposit Contracts:

$$c_1^* = 1 \quad c_{2h}^* = 1.3 \quad c_{2l}^* = 0.3$$

$$E(U) = 0.1165$$

*Proof.* See Proof I in Appendix A

Table 2: No Equity Capital: Banks' Optimal Balance Sheet Structure and Repayments to Depositors

Without having the opportunity to raise equity capital, banks invest their total amount of deposits in the short-term asset and in government bonds i.e., only in liquid assets. They do not grant loans.<sup>11</sup> Basically, investing in loans has two effects on consumers' consumption: First, it increases expected consumption at date 2 as the expected loan return is higher than that of government bonds. Second, it decreases consumption at date 1 as, due to the budget constraint (11), an increase in loan investment implies a respective decrease of investment in liquid assets, and early consumers are only repaid with the proceeds of the liquid assets. In our numerical example, the effects of the loans' illiquidity on consumption is so strong that even at point  $u = 0$  the marginal utility from date-1 consumption exceeds the expected marginal utility from date-2 consumption, i.e. the non-negativity constraint on  $u$  becomes binding.

Moreover, banks divide their investment equally into the liquid assets,  $x^* = y^*$ . With respect to this result two aspects are important. First, one half of the banks are early banks whereas the other half are late banks. Second, there is no aggregate liquidity uncertainty, so that at date 0, banks know the aggregate supply and demand in the government bond market for date 1 and therefore the equilibrium price  $p^{**} = 1$  (see Section 4 for details).

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<sup>11</sup>Diamond and Dybvig (1983) consider the explicit role of banks in an economy in the sense that banks transform illiquid assets into liquid liabilities i.e. banks allow better risk-sharing among consumers with different consumption preferences. However, in our numerical example without equity capital, banks do not invest in illiquid loans i.e. banks are obsolete in this case. However, with the introduction of equity capital, banks start to invest in illiquid assets and create liquidity, i.e. there is a role for banks in our subsequent analyses.

Accordingly, all banks invest the identical amount in government bonds and in the short-term asset, to be able to hedge their idiosyncratic liquidity risks completely by trading government bonds on the interbank market at date 1 when consumption uncertainty is resolved. This allows us to set  $x^* = y^* = 0.5z^*$  in our subsequent analyses. The variable  $z^*$  thus donates a bank's optimal investment in liquid assets (short-term asset and government bonds).

## 5.2 With Equity Capital

If banks have the opportunity to raise equity capital from investors, but do not face a binding minimum capital ratio ( $CR^{min} = 0$ ), we will get the solutions given in Table 3 for optimal bank behaviour. The results show that even if the banking sector is not

Balance Sheet					
A			L		
$x^*$	0.4544	41.87%	$e_0^*$	0.0853	7.86%
$y^*$	0.4544	41.87%	$D$	1	92.14%
$u^*$	0.1765	16.26%			
$\sum$	1.0853	100%	$\sum$	1.0853	100%

Contracts with Investors:

early banks:	$e_{2H}^*=0.2718$	$e_{2L}^*=0.0441$		
late banks:	$e_{2Hh}^*=0$	$e_{2Hl}^*=0$	$e_{2Lh}^*=0$	$e_{2Ll}^*=0$

Deposit Contracts:

$$c_1^* = 0.9088 \quad c_{2Hh}^* = 1.4532 \quad c_{2Hl}^* = 0.5444 \quad c_{2Lh}^* = 1.2256 \quad c_{2Ll}^* = 0.3168$$

$$E(U) = 0.1230$$

*Proof.* See Proof II in Appendix A

Table 3: With Equity Capital: Banks' Optimal Balance Sheet Structure and Repayments to Investors and Depositors

subject to capital requirements, it will be optimal for banks to raise equity capital. Since  $\rho > E(K)$ , equity capital is costly for banks in the sense that investors' opportunity costs, and thus the amount banks expect to repay to investors, exceed the expected return even of the banks' most profitable asset – in our case, loans. Nevertheless, banks have an

incentive to raise equity capital as it allows the liquidity risk involved with an investment in relatively high profitable loans to be transferred from risk-averse depositors to risk-neutral investors, leading to an increase in depositors' expected utility ( $E(U)|_{no\ capital} < E(U)|_{with\ capital, CR^{min}=0}$ , see tables 2 and 3).

A crucial point is that with the possibility to raise equity, the banks' budget constraint is softened. In the case without this possibility (Section 5.1), an increase in loan investment leads to a decrease of investment in liquid assets to the same amount, ( $\frac{\partial z}{\partial u}|_{no\ capital} = -1$ ), and thus to a respective decline in date-1 consumption. However, with the possibility of raising equity capital, an increase in loans leads to a lower necessary decrease in liquid assets,  $\frac{\partial z}{\partial u}|_{with\ capital, CR^{min}=0} > -1$ .<sup>12</sup> Consequently, an investment in relatively highly profitable but illiquid loans, which increases expected date-2 consumption, only implies a relatively small decrease of consumption at date 1, so that there is an increase in depositors' expected utility.<sup>13</sup> It is crucial that a huge part of the additional loan investment is financed by raising equity capital from risk-neutral investors. Due to their risk-neutrality it is optimal that they bear the liquidity risk involved with the banks' loan investment. The risk-averse depositors' thus benefit from a liquidity risk transfer to the risk-neutral investors.

Optimal risk-sharing implies that if it turns out that a bank is an early bank, the investors of this bank will receive the total proceeds from the loan investment at date 2 ( $e_{2H}^*, e_{2L}^* > 0$ ). If it turns out that a bank is a late bank, they will receive nothing ( $e_{2Hh}^*, e_{2Hl}^*, e_{2Lh}^*, e_{2Ll}^* = 0$ ). Considering investors thus get repaid with the total proceeds from the bank loan investment but only with probability 0.5, and that their opportunity costs are higher than the expected return on loans ( $\rho > E(K)$ ), the bank loan investment must exceed the amount of raised equity capital to be able to satisfy investors' claims. This means that it is not possible to finance an additional loan investment exclusively by raising more equity, i.e. an increase in loan investment is still associated with a decrease of investment in liquid assets ( $-1 < \frac{\partial z}{\partial u}|_{with\ capital, CR^{min}=0} < 0$ ). Formally, the

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<sup>12</sup>In our numerical example  $\frac{\partial z}{\partial u}|_{with\ capital, CR^{min}=0} = -0.5168 > -1$ .

<sup>13</sup>Formally, with the possibility of raising equity capital, at point  $u = 0$  the expected marginal utility from date-2 consumption exceeds the marginal utility from date-1 consumption, and it is optimal for banks to increase their investment in loans relative to their investment in liquid assets. This bank behaviour increases depositors' expected date-2 consumption and decreases their date-1 consumption and thus balances the marginal utilities.

investors' incentive-compatibility constraint given by (9) becomes  $e_0^* \rho = 0.5 u^* E(K)$ , so that  $\frac{2\rho}{E(K)} = \frac{u^*}{e_0^*} |_{CR^{min}=0}$ . This means that the loan investment needs to be at least  $\frac{2\rho}{E(K)}$  times higher than the amount of raised equity capital. The expression shows that the optimal investment in loans relative to the raised equity capital must be higher the lower the expected returns are of the loans compared to the investors' opportunity costs.<sup>14</sup> In our numerical example  $\frac{2\rho}{E(K)} = \frac{u^*}{e_0^*} |_{CR^{min}=0} = 2.0692$ .

### 5.3 Binding Capital Requirement for Loans

In this section, we analyse bank behaviour when banks face a required minimum capital ratio with a preferential treatment of risky sovereign bonds in the sense that only risky loans have to be backed with equity. We introduce a risk weight of 1 for this asset class, i.e. in constraint (10) we have  $\phi_x = \phi_y = 0$  and  $\phi_u = 1$ . In our analysis, we suppose a binding required minimum capital ratio  $CR^{min} |_{\phi_u=1, \phi_x=\phi_y=0} = \frac{e_0}{u} = 0.5799$ .<sup>15</sup> The results for optimal bank behaviour under this additional constraint are shown in Table 4. The comparison of the results for optimal bank behaviour given in tables 3 and 4 reveals that the capital requirement induces banks to raise more equity and to increase their investments in loans and liquid assets ( $e_0^*$ ,  $u^*$ , and  $z^*$  increase). In the following, we will have a closer look at this bank reaction. We divide our analysis of the banks' adjustment behaviour into two steps. In the first step, we look at the direct consequences of the introduced capital regulation for bank investment and financing behaviour and thus for depositors' consumption possibilities. In a second step, we then analyse the banks' optimal response to these changed consumption possibilities.

First step: In Section 5.2 it has been shown that without capital requirements an investor will only get repaid if it turns out that his bank is an early bank. Then, at date 2, he receives the total proceeds from his bank's loan investment. However, the capital

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<sup>14</sup>In our model, the probability of becoming an early bank is  $(1 - \omega = 0.5)$ , see Section 3. Not inserting 0.5 for  $1 - \omega$ , we have  $\frac{\rho}{(1-\omega)E(K)} = \frac{u^*}{e_0^*} |_{CR^{min}=0}$  which reveals that investment in loans relative to the raised equity capital must also be the higher the lower the probability of becoming an early bank is, i.e. the lower the probability is that the investor actually gets repaid.

<sup>15</sup>If banks do not face binding capital requirements (Section 5.2) they choose an optimal capital ratio of:  $CR^{opt} = \frac{e_0^*}{u^*} = \frac{0.0853}{0.1765} = 0.4833$ . In order to analyse the impact of a binding capital ratio,  $CR^{min} > CR^{opt}$  must hold. We consider a binding minimum capital ratio which is 20% higher than  $CR^{opt}$ .

Balance Sheet

A			L		
$x^*$	0.4581	41.06%	$e_0^*$	0.1157	10.37%
$y^*$	0.4581	41.06%	$D$	1	89.63%
$u^*$	0.1995	17.88%			
$\sum$	1.1157	100%	$\sum$	1.1157	100%

Contracts with Investors:

early banks:	$e_{2H}^*=0.3072$	$e_{2L}^*=0.0499$			
late banks:	$e_{2Hh}^*=0.0635$	$e_{2Hl}^*=0$	$e_{2Lh}^*=0$	$e_{2Ll}^*=0$	

Deposit Contracts:

$$c_1^* = 0.9162 \quad c_{2Hh}^* = 1.4348 \quad c_{2Hl}^* = 0.5821 \quad c_{2Lh}^* = 1.2409 \quad c_{2Ll}^* = 0.3247$$

$$E(U) = 0.1224$$

*Proof.* See Proof III in Appendix A

Table 4: Binding Capital Requirement for Loans: Banks' Optimal Balance Sheet Structure and Repayments to Investors and Depositors

regulation-induced decrease of  $\frac{u}{e_0}$  implies that this is no longer possible.<sup>16</sup> The decreased loan-to-equity ratio implies that the expected returns from loans are no longer sufficient to satisfy investors' claims. In Section 5.2 we have shown that to have the expected returns from loans to be sufficient to satisfy investors' claims, requires a loan investment to be at least  $\frac{2\rho}{E(K)} = \frac{u^*}{e_0^*}|_{CR^{min}=0}$  times higher than the raised equity capital. However,  $\frac{u^*}{e_0^*}|_{CR^{min}|\phi_u=1, \phi_x=\phi_y=0} < \frac{u^*}{e_0^*}|_{CR^{min}=0}$ .<sup>17</sup> Consequently, with the capital requirement not only early banks but also late banks have to pay a positive amount to their investors ( $e_{2Hh}^*|_{CR^{min}=0} = 0$  but  $e_{2Hh}^*|_{CR^{min}|\phi_u=1, \phi_x=\phi_y=0} > 0$ ). This reduces depositors' expected date-2 consumption as they have to share the returns from the long-term assets with the investors. Moreover, the budget constraint implies that the required decrease in  $\frac{u}{e_0}$  leads to an increase in bank investment in liquid assets ( $z$  increases). This increases depositors' date-1 consumption. Accordingly, in the first step of our analysis the introduced capital

<sup>16</sup>Note that the required loan-to-equity ratio is the reciprocal of the minimum capital ratio ( $\frac{u}{e_0}|_{CR^{min}|\phi_u=1, \phi_x=\phi_y=0} = \frac{1}{CR^{min}|\phi_u=1, \phi_x=\phi_y=0}$ ).

<sup>17</sup>In our numerical example  $\frac{u^*}{e_0^*}|_{CR^{min}|\phi_u=1, \phi_x=\phi_y=0} = 1.7243 < \frac{u^*}{e_0^*}|_{CR^{min}=0} = 2.0692$ .

requirement for loans implies that the expected marginal utility from date-2 consumption exceeds the marginal utility from date-1 consumption.

Second step: To remove this inefficiency, banks increase loans and equity according to the required minimum loan-to-equity ratio  $\frac{u}{e_0}|_{CR^{min}|\phi_u=1,\phi_x=\phi_y=0}$ . As this ratio is larger than one (in our numerical example it is equal to 1.7243), the balance sheet constraint implies that investment in liquid assets decreases again. This second-step bank behaviour leads to a higher (lower) date-2 consumption (date-1 consumption) until the marginal utility from date-1 consumption again equals the expected marginal utility from date-2 consumption. Since the increase in liquid assets in the first step exceeds the decrease in the second step, in absolute terms there is an overall increase in liquid assets.

Consequently, the introduction of the binding required minimum capital ratio implies that the potential for liquidity risk transfer can no longer be fully exploited,  $\frac{u^*}{e_0^*}|_{CR^{min}|\phi_u=1,\phi_x=\phi_y=0} < \frac{u^*}{e_0^*}|_{CR^{min}=0}$ , the banks' loan-to-liquid asset ratio increases,  $\frac{u^*}{z^*}|_{CR^{min}|\phi_u=1,\phi_x=\phi_y=0} > \frac{u^*}{z^*}|_{CR^{min}=0}$ . The restricted possibility of transferring liquidity risk from depositors to investors leads to a lower depositors' expected utility,  $E(U)|_{CR^{min}=0} > E(U)|_{CR^{min}|\phi_u=1,\phi_x=\phi_y=0}$ .

## 5.4 Binding Capital Requirements for Loans and Government Bonds

This section analyses bank optimal investment and financing behaviour when the required minimum capital ratio also includes a positive risk weight for government bond holdings, i.e. when risky sovereign exposures are also subject to capital regulation. We assume a risk weight for government bonds of  $\phi_y = 0.05$ .<sup>18</sup> The risk weights for loans and the short-term asset are the same as in Section 5.3, i.e.  $\phi_u = 1$  and  $\phi_x = 0$ . Also the level of the required minimum capital ratio is not changed,  $CR^{min} = 0.5799$ . Hence, the capital regulation constraint becomes  $CR^{min}|_{\phi_u=1,\phi_y=0.05,\phi_x=0} = \frac{e_0}{u+0.05y} = 0.5799$ . The resulting optimal bank behaviour is illustrated in Table 5.

Comparing the results given in tables 4 and 5 shows that repealing the preferential treatment of government bonds in capital regulation induces banks to raise additional equity capital, to grant more loans and to increase their liquid asset holdings ( $e_0^*$ ,  $u^*$ , and  $z^*$  increase). Consequently, the bank reaction is the same as when a binding capital ratio

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<sup>18</sup>By assuming that  $\phi_u = 1 > \phi_y = 0.05$ , we consider sovereign bonds to be less risky than loans.

Balance Sheet

A			L		
$x^* =$	0.4586	40.02%	$e_0^* =$	0.1459	12.73%
$y^* =$	0.4586	40.02%	$D =$	1	87.27%
$u^* =$	0.2287	19.96%			
$\Sigma$	1.1459	100%	$\Sigma$	1.1459	100%

Contracts with Investors:

early banks:	$e_{2H}^* = 0.3522$	$e_{2L}^* = 0.0572$		
late banks:	$e_{2Hh}^* = 0.1166$	$e_{2Hl}^* = 0$	$e_{2Lh}^* = 0$	$e_{2Ll}^* = 0$

Deposit Contracts:

$$c_1^* = 0.9172 \quad c_{2Hh}^* = 1.4281 \quad c_{2Hl}^* = 0.6274 \quad c_{2Lh}^* = 1.2496 \quad c_{2Ll}^* = 0.3323$$

$$E(U) = 0.1217$$

*Proof.* See Proof IV in Appendix A

Table 5: Capital Requirements for Loans and Government Bonds: Banks' Optimal Balance Sheet Structure and Repayments to Investors and Depositors

is introduced only for loans, as described in Section 5.3. Also, the reason behind this reaction is the same: if banks also have to back government bonds with equity, they can invest less in highly profitable loans per units of costly equity capital,  $\frac{u^*}{e_0^*}$  decreases.<sup>19</sup>

Accordingly, as in the case in which only loans have to be backed with equity, the expected returns from loans are not sufficient to satisfy investors' claims, so that not only early but also late banks have to make a positive payment to the investors. If also, government bonds have to be backed with equity, this payment must be even higher because of the lower loan-to-equity ratio  $\frac{u^*}{e_0^*}$ , i.e.  $e_{2Hh}^*|_{CR^{min}=0} = 0 < e_{2Hh}^*|_{CR^{min}|_{\phi_u=1, \phi_x=\phi_y=0}} < e_{2Hh}^*|_{CR^{min}|_{\phi_u=1, \phi_y=0.05, \phi_x=0}}$ . Consequently, with respect to the optimal bank investment and financing behaviour the same adjustment process as described in detail in Section 5.3 applies. This means that the potential to transfer liquidity risk from depositors to investors will be used even less,  $\frac{u^*}{e_0^*}|_{CR^{min}|_{\phi_u=1, \phi_y=0.05, \phi_x=0}} <$

<sup>19</sup>Given that  $CR^{min} = \frac{e_0}{u+0.05y}$ , the positive risk weight for sovereign bonds implies for  $y^* > 0$  that  $\frac{u^*}{e_0^*}|_{CR^{min}|_{\phi_u=1, \phi_y=0.05, \phi_x=0}} < \frac{u^*}{e_0^*}|_{CR^{min}|_{\phi_u=1, \phi_x=\phi_y=0}}$ . In our numerical example  $\frac{u^*}{e_0^*}|_{CR^{min}=0} = 2.0692 > \frac{u^*}{e_0^*}|_{CR^{min}|_{\phi_u=1, \phi_x=\phi_y=0}} = 1.7243 > \frac{u^*}{e_0^*}|_{CR^{min}|_{\phi_u=1, \phi_y=0.05, \phi_x=0}} = 1.5675$ .



$\frac{u^*}{e_0^*}|_{CR^{min}|\phi_u=1, \phi_x=\phi_y=0} < \frac{u^*}{e_0^*}|_{CR^{min}=0}$ , the banks' loan-to-liquid asset ratio increases further,  $\frac{u^*}{z^*}|_{CR^{min}|\phi_u=1, \phi_y=0.05, \phi_x=0} > \frac{u^*}{z^*}|_{CR^{min}|\phi_u=1, \phi_x=\phi_y=0} > \frac{u^*}{z^*}|_{CR^{min}=0}$ . The stronger restriction of transferring liquidity risk from depositors to investors further reduces the depositors' expected utility,  $E(U)|_{CR^{min}=0} > E(U)|_{CR^{min}|\phi_u=1, \phi_x=\phi_y=0} > E(U)|_{CR^{min}|\phi_u=1, \phi_y=0.05, \phi_x=0}$ .

## 6 Financial Stability

The aim of this paper is to analyse the resilience of the banking sector in case of a sovereign debt crisis under different capital regulation scenarios. This section shows that increasing doubts about sovereign solvency may lead to liquidity issues in the banking sector driven by a respective price drop for sovereign bonds. A central bank acting as a LOLR can avoid bank insolvencies due to liquidity issues. It turns out that in the presence of a LOLR the abolishment of the preferential treatment of sovereign bonds in financial regulation strengthens the resilience of the banking sector in the case of a sovereign debt crisis.

### 6.1 Government Bond Shock

After the banks have made their financing and investment decisions at date 0, but before the start of interbank trading at date 1, the economy is hit by a shock in the form of an increase in the default probability of government bonds (we refer to this shock as a government bond shock). This implies a respective decrease of the expected return on government bonds. Denoting after-shock variables with a bar, we thus have  $(1 - \bar{\beta}) > (1 - \beta)$  and  $E(S) > \bar{E}(S)$ . As the liquidity shock in Allen and Gale (2000), this government bond shock is assigned a zero probability at date 0, when investment decisions are made. The expected return on the loan portfolio and the return on the short-term asset are not affected by the shock.<sup>20</sup>

The shock influences the late banks' demand for sovereign bonds in the interbank market at date 1. The decline in the expected return on government bonds implies that the maximum price late banks are willing to pay for a bond decreases (equations (15)

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<sup>20</sup>To keep the model as simple as possible, we assume that the expected loan return is not affected by the government bond shock. However, there is empirical evidence that there are spillovers going from sovereigns to other sectors of an economy (see e.g. Corsetti et al. (2013)) as sovereigns' ratings normally apply as a "sovereign floor" for the ratings assigned to private borrowers. Nevertheless, if we take this correlation into account our results will not qualitatively change. See footnote 26 for details.

and (16)). The early banks' supply of government bonds is not affected by the shock. As their depositors only value consumption at date 1, they want to sell their total holdings of government bonds, independent of their default probability (see equation (17)).

To be able to satisfy their depositors according to the deposit contract, the price an early bank receives for a government bond must be at least one, i.e. we have a critical price

$$p^{crit} = 1. \quad (19)$$

Setting in equation (15)  $p^{max}$  equal to  $p^{crit}$  and then solving the equation for  $(1 - \beta)$  gives us the critical default probability

$$(1 - \beta^{crit}) = \frac{\ln(h) - \ln(p^{crit})}{\ln(h) - \ln(l)} = \frac{\ln(h)}{\ln(h) - \ln(l)}. \quad (20)$$

If the after-shock default probability of government bonds exceeds this critical probability, the expected return on government bonds will become so low that the maximum price late banks are willing to pay for a bond will fall below one, early banks will be illiquid and insolvent. The threshold  $(1 - \beta^{crit})$  allows us to distinguish between a small  $(1 - \overline{\beta^{small}}) \leq (1 - \beta^{crit})$  and a large government shock  $(1 - \overline{\beta^{large}}) > (1 - \beta^{crit})$ . In the following, we will comment on the consequences of both shocks in more detail.

### Small Government Bond Shock

Figure 3 illustrates the interbank market for a small government bond shock. The increased sovereign default probability induces that the maximum price late banks are willing to pay for sovereign bonds decreases,  $\overline{p^{max\ small}} < p^{max}$ . However, the equilibrium price and the equilibrium transaction volume after a small shock do not change,  $\overline{p^{**small}} = p^{**} = 1$ ,  $\overline{y^{**small}} = y^{**} = 0.5y^*$ . Consequently, the small shock only implies a decline in the late banks' surplus from interbank trading as at the same price late banks receive the same quantity of government bonds but they yield a lower expected return. This is illustrated by the blue shaded area in Figure 3. The early banks' surplus does not change as their

depositors value consumption only at date 1 so that for them the decreased expected (date-2) return on government bonds plays no role.

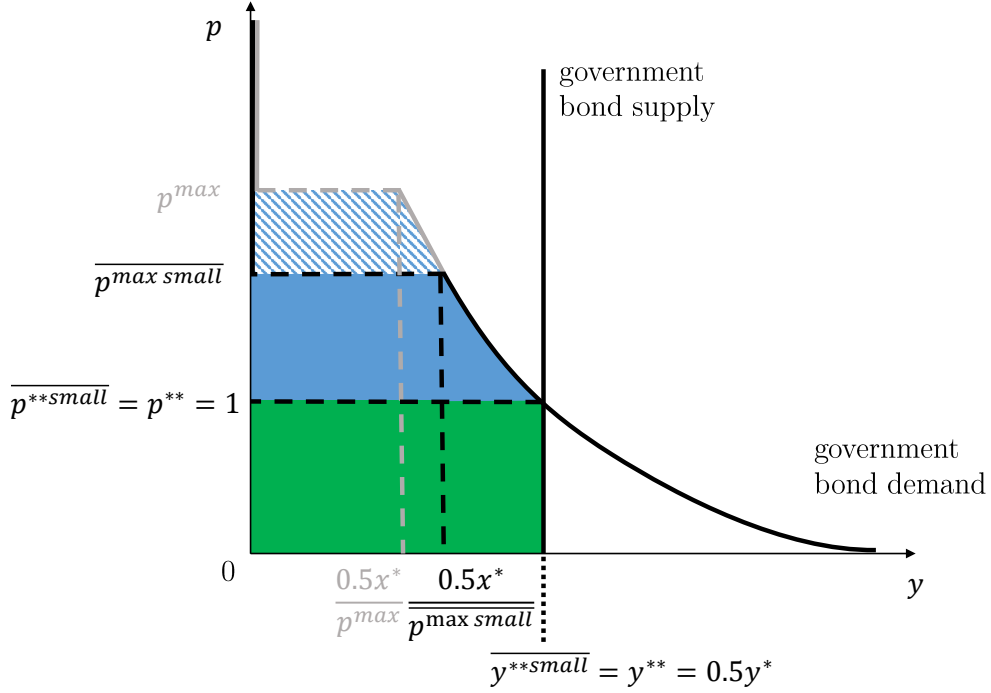


Figure 3: Interbank Market for Government Bonds at Date 1;  $(1 - \overline{\beta^{small}}) \leq (1 - \beta^{crit})$

In the following, we will discuss in more detail who actually bears the costs of a small government bond shock. Early-bank depositors are not affected by the shock as there is neither a shock-induced change in the equilibrium price, nor in the equilibrium transaction volume on the interbank market for government bonds, so that their consumption does not change:

$$\overline{c_1^{small}} = x^* + y^* \overline{p^{**small}} = x^* + y^* p^{**} = x^* + y^* = c_1^*. \quad (21)$$

Early-bank investors are not affected by the shock either as they are only repaid from the proceeds of the loan portfolio. However, the shock influences late-bank depositors as due to the decreased expected return on government bonds their expected date-2 consumption decreases.<sup>21</sup> Whether late-bank investors are affected depends on whether there is a

<sup>21</sup>Note, that nevertheless late banks do not become insolvent as they can still fulfil the contracts with their depositors as the contractually agreed repayments are not influenced by the shock,  $\overline{c_{2(\cdot)}^{small}} = c_{2(\cdot)}^*$  (see equations (5)-(8)).

binding capital requirement. If there is no binding capital requirement, the shock will not impact late-bank investors as then, independent of the shock, their repayment will anyhow be equal to zero (see Section 5.2). However, binding capital requirements imply that if both government bonds and the loan portfolio succeed (state  $Hh$ ), also late-bank investors will get some repayment at date 2 (see Section 5.3 and 5.4). As the shock implies that the occurrence probability of this event becomes smaller, their expected date-2 repayment decreases.

### Large Government Bond Shock

Figure 4 illustrates the interbank market for a large government bond shock. The increase in the government bonds' default probability is so high that their expected return becomes so low that the maximum price late banks are willing to pay for a bond falls below one. Considering equation (15), the after-shock equilibrium price thus becomes

$$\overline{p^{**large}} = \overline{p^{max\ large}} < 1. \quad (22)$$

At the equilibrium price  $\overline{p^{**large}}$ , there is an excess demand for government bonds<sup>22</sup> but the equilibrium trading volume has not changed,  $\overline{y^{**large}} = y^{**} = 0.5y^*$ . As  $\overline{p^{max\ large}} = \overline{p^{**large}}$  the late-banks' surplus from interbank trading becomes zero. In addition, the fall of the equilibrium price ( $\overline{p^{**large}} < p^{**}$ ) also leads to a decrease of the early banks' surplus from interbank trading. They receive a smaller quantity of the short-term asset in exchange for their total holdings of government bonds. The decrease of the equilibrium price below 1 means that early banks are no longer able to fulfil their deposit contracts:

$$\overline{c_1^{large}} = x^* + y^* \overline{p^{**large}} < x^* + y^* p^{**} = x^* + y^* = c_1^*. \quad (23)$$

Early banks are thus insolvent and are liquidated at date 1. In contrast to the small government bond shock, depositors and investors of both early and late banks are affected by the large shock. Early-bank depositors suffer as their date-1 consumption decreases

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<sup>22</sup>The reason is that late banks want to sell their total holdings of the short-term asset ( $0.5x^*$ ) in exchange for government bonds. However, the supply of government bonds is limited to the early banks' total holdings of this asset ( $0.5y^*$ ), so that at prices below 1, there is an excess demand.

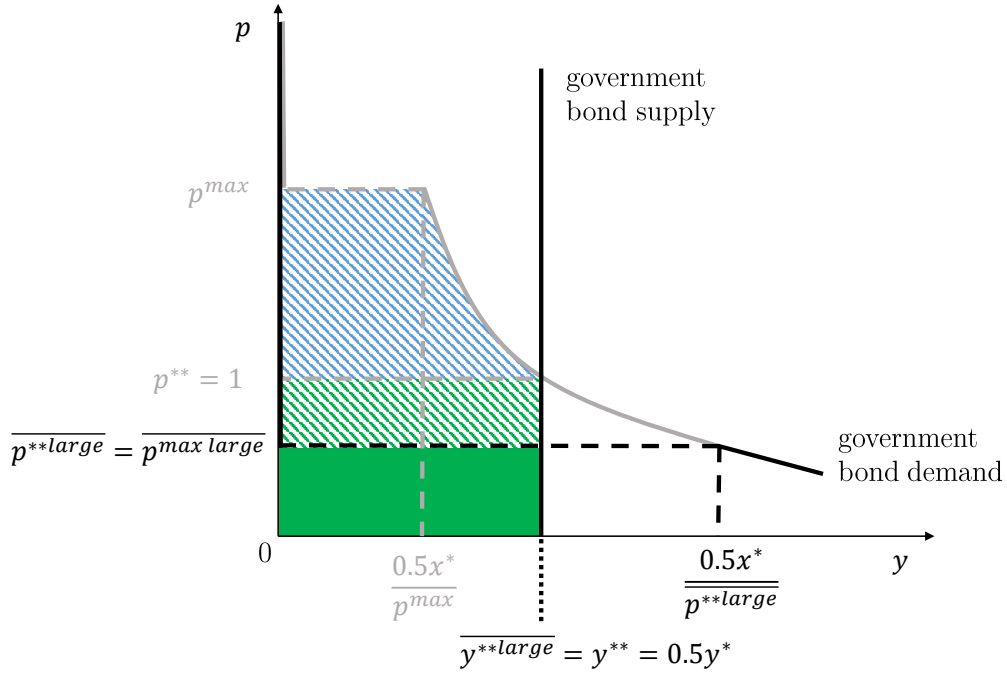


Figure 4: Interbank Market for Government Bonds at Date 1;  $(1 - \overline{\beta^{large}}) > (1 - \beta^{crit})$

$(c_1^{large} < c_1^*)$  and early-bank investors suffer as the loan portfolio's liquidation value at date 1 is zero, so that early-bank investors get no repayment at all.<sup>23</sup> With respect to the late-bank depositors and investors the same argument as in the small-shock scenario holds. Depositors are affected by the shock as their expected date-2 consumption decreases due to the decreased expected return on government bonds. Investors will be affected by the shock if there is a binding capital ratio, as only then may they be repaid, but the probability of actually being repaid declines.

## 6.2 Central Bank as a Lender of Last Resort

As banks hold government bonds in order to hedge their idiosyncratic liquidity risks, a government bond price drop may lead to liquidity issues for banks and thus to insolvencies. To avoid bankruptcies due to liquidity issues we introduce a central bank as a lender of last resort (LOLR) in the sense of Bagehot (1873). The central bank provides liquidity

<sup>23</sup>If one assumes that a liquidator keeps the loan portfolio until date 2, so that a positive return on the portfolio is realised (either  $H$  or  $L$ ), it does not prevent the early banks from going bankrupt at date 1. However, the early-bank investors would not be affected by the shock. Note that it is not possible to let the investors bear the costs of the shock instead of the depositors by paying them the proceeds of the loan portfolio at date 2 as they only value consumption at date 1.

to troubled banks against adequate collateral. In our model, banks' loan portfolios serve as collateral. In order to avoid any potential losses for the central bank, the maximum amount of liquidity  $\psi$  the central bank is willing to provide to an early bank against its loan portfolio as collateral is<sup>24</sup>

$$\psi = u^* L. \quad (24)$$

An early bank's additional liquidity needs after a large government bond shock  $\tau$  are determined by the repayment agreed upon in the deposit contract  $c_1^*$  and the lower after-shock repayment  $\overline{c_1^{large}}$  (without a LOLR):

$$\tau = c_1^* - \overline{c_1^{large}} = y^*(p^{**} - \overline{p^{**large}}) = y^*(1 - \overline{p^{**large}}). \quad (25)$$

Equation (25) reveals that the bank's additional liquidity needs increase in its holdings of government bonds  $y^*$  and in the extent of the shock which is reflected by the decrease of the government bond price  $(p^{**} - \overline{p^{**large}})$ . The promised repayment to early consumers  $c_1^*$  increases in a bank's holdings of government bonds ( $c_1^* = x^* + y^* p^{**} = x^* + y^*$ ). The shock-induced price drop for government bonds below one therefore implies the additional liquidity needs the larger the higher the bank's holdings of government bonds  $y^*$  are.

The comparison of a bank's additional liquidity needs  $\tau$  with the maximum amount of liquidity the central bank is willing to provide  $\psi$  gives us the critical government bond price

$$p^{critLOLR} = 1 - \frac{u^* L}{y^*} < 1. \quad (26)$$

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<sup>24</sup>Considering that potential interest payments for the additional central bank liquidity should also be covered by collateral, does not qualitatively change our results. In that case, the maximum amount of liquidity  $\psi$  the central bank is willing to provide against the loan portfolio as collateral decreases. This decrease implies that the shock-absorbing capacity of the banking sector in the presence of a LOLR ( $SAC^{LOLR}$ ) becomes smaller in both regulation scenarios i.e. with and without a preferential regulatory treatment of government bonds. However, as the loan-to-liquid assets ratio  $\frac{u^*}{\tilde{R}^*}$  is higher without a preferential treatment of sovereign debt in bank capital regulation, the  $SAC^{LOLR\tilde{R}^*}$  will be higher in this case (see equation (30)).

Inserting  $p^{critLOLR}$  for  $\overline{p^{**large}}$  in equation (22) and then solving the equation for  $(1 - \overline{\beta^{large}})$ , gives us the critical default probability

$$(1 - \beta^{critLOLR}) = \frac{\ln(h) - \ln(p^{critLOLR})}{\ln(h) - \ln(l)} = \frac{\ln(h) + \ln(\frac{u^*L}{y^*})}{\ln(h) - \ln(l)} = \frac{\ln(h) + \ln(\frac{u^*}{z^*}2L)}{\ln(h) - \ln(l)}. \quad (27)$$

If the government bond shock is so large that  $(1 - \overline{\beta^{large}}) > (1 - \beta^{critLOLR})$ , the equilibrium price  $\overline{p^{**large}}$  will fall below  $p^{critLOLR}$ , and early banks will become insolvent, despite the existence of a LOLR. The reason is that the central bank is only willing to provide liquidity to illiquid but not to insolvent banks.<sup>25</sup> The liquidity issue leads to a solvency issue as the price drop, and thus the resulting early banks' liquidity problem, will be so huge that they will not have sufficient collateral to obtain enough liquidity from the LOLR.

Comparing the critical default probability with and without a LOLR (see equations (20) and (27)) reveals that with a LOLR the critical default probability is higher. However, the comparison also shows that with a LOLR the critical default probability does not only depend on the possible government bond returns  $h$  and  $l$ , as is the case without a LOLR, but, in addition, on the loan portfolio return  $L$  and the bank's investment in government bonds  $y^*$  and loans  $u^*$ . An increase in  $y^*$  leads to a decrease of the critical default probability as then an early bank needs more additional liquidity after a government bond shock (see equation (25)). The critical default probability increases in  $u^*$  and  $L$ , as then an early bank's collateral increases in quantity and value, so that in case of a shock it can obtain more additional liquidity from the central bank (see equation (24)).<sup>26</sup> This has important implications for the banking sector's shock-absorbing capacity under the different capital regulation approaches as we will see in Section 6.3.

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<sup>25</sup>Even if one assumes that the central bank cannot distinguish between illiquid and insolvent banks, the main results do not change. Providing liquidity to insolvent banks does not prevent their insolvency as the maximum liquidity the central bank is willing to provide will not be sufficient to cover the additional liquidity needs of insolvent early banks ( $\tau > \psi$ ).

<sup>26</sup>We argued at the beginning of this section that considering a possible spillover of the government bond shock to loans would not lead to a qualitative change of our results. In case the probability of loan success is negatively affected by the government bond shock, i.e. if  $\alpha > \bar{\alpha}$ , the expected consumers' consumption at date 2 will decrease. However, there are no liquidity issues for late banks as the contractually agreed repayments to the consumers are not influenced. The crucial point is that the potential increase in  $\alpha$  neither induces a change in the liquidity provision by the central bank ( $\psi$ ) nor does it lead to an additional liquidity demand ( $\tau$ ). As these variables determine the shock-absorbing capacity with a LOLR (see Section 6.3), spillover effects from sovereign to loans have no impact on our results.

If the central bank acts as a lender of last resort and if  $(1 - \beta^{crit}) < (1 - \overline{\beta^{large}}) \leq (1 - \beta^{critLOLR})$ , early-bank depositors are not affected by the shock, their consumption does not change:

$$\overline{c_1^{large}} = x^* + y^* \overline{p^{**large}} + \tau = x^* + y^* = c_1^* \quad (28)$$

Early-bank investors, however, are affected by the shock as they get repaid from the proceeds of the loan portfolio and a part of these proceeds has to be used to repay the central bank. With respect to the late-bank depositors and investors the same arguments as in the scenarios without a LOLR hold. Depositors are affected by the shock as their expected date-2 consumption decreases. Investors will be affected if there is a binding capital requirement.<sup>27</sup> However, if  $(1 - \overline{\beta^{large}}) > (1 - \beta^{critLOLR})$  early banks are insolvent and thus the central bank does not provide liquidity. Then depositors and investors of both early and late banks are affected by the shock and the identical arguments hold as in the large-shock scenario without a LOLR.

### 6.3 The Shock Absorbing Capacity of the Banking Sector in Different Capital Regulation Scenarios

The above analysis allows us to discuss the (government bond) shock-absorbing capacity of the banking sector, and in this sense its stability,<sup>28</sup> in two different capital regulation scenarios. The difference between the critical and the initial default probability of government bonds serves as a measure of the banking sector's shock-absorbing capacity. The measure thus shows how large a government bond shock can be without implying the insolvency of early banks and thus of a huge part of the banking sector. Considering equa-

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<sup>27</sup>Note that again late-bank depositors are *only* affected by the shock due to the decreased expected return on government bonds. Late banks do not borrow any additional liquidity from the central bank so that they do not have to use part of the proceeds from the loan portfolio to repay the central bank.

<sup>28</sup>The ECB defines financial stability as a condition in which the financial system – intermediaries, markets and market infrastructures – can withstand shocks without major distribution in financial intermediation and the general supply of financial services.



tions (20) and (27) and denoting the shock-absorbing capacity by  $SAC$  and  $SAC^{LOLR}$  respectively, we get

$$SAC = (1 - \beta^{crit}) - (1 - \beta) = \frac{\ln(h)}{\ln(h) - \ln(l)} - (1 - \beta) \quad (29)$$

for the banking sector's shock absorbing capacity without a LOLR and

$$SAC^{LOLR} = (1 - \beta^{critLOLR}) - (1 - \beta) = \frac{\ln(h) + \ln(\frac{u^*}{z^*} 2L)}{\ln(h) - \ln(l)} - (1 - \beta) \quad (30)$$

for the banking sector's shock absorbing capacity with a LOLR.

Equation (29) reveals that without a LOLR, the shock-absorbing capacity is not at all influenced by capital requirements. The reason is that without a LOLR early banks will become insolvent if the maximum price for government bonds that late banks are willing to pay. Then, early banks will no longer be able to satisfy their customers according to the deposit contract. However, the maximum price late banks are willing to pay is only determined by the expected return on a government bond (see equation (15)) which will not change if capital requirements are introduced. Hence, if there is no LOLR, the shock-induced liquidity problem cannot be solved by any kind of capital requirements, the difference  $(1 - \beta^{crit}) - (1 - \beta) = SAC$  is always the same. This result is illustrated in Figure 5 by the solid line.

However, as shown by (30), with a LOLR binding capital requirements influence the banking sector's shock-absorbing capacity. The reason is that binding capital requirements influence bank investment behaviour. They imply an increase in both government bond investments  $y^*$  and loan investments  $u^*$ . The former implies an increase in the banks' additional liquidity needs  $\tau$  after the shock (see equation (25)), and thus lowers the shock-absorbing capacity. The latter leads to an increase in the additional liquidity  $\psi$  the central bank is willing to provide, and therefore also in the shock absorbing capacity. However, the increase in  $u^*$  is stronger than in  $y^*$  (and thus than in  $z^*$ , see Section 5) which implies that there is an overall increase in the  $SAC^{LOLR}$ . The main driver for this result is that the regulation-induced increase in  $u^*$  implies that banks have more collateral to obtain additional liquidity from the central bank after a respective shock.

As the increase in  $\frac{u^*}{z^*}$  will be higher if not only loans but also sovereign bonds have to be backed with equity ( $\frac{u^*}{z^*}|_{CR^{min}|\phi_u=1,\phi_y=0.05,\phi_x=0} > \frac{u^*}{z^*}|_{CR^{min}|\phi_u=1,\phi_x=\phi_y=0} > \frac{u^*}{z^*}|_{CR^{min}=0}$ ), the (government bond) shock-absorbing capacity of the banking sector will be the highest if there is a LOLR and if loans and sovereign bonds have to be backed with equity ( $SAC^{LOLR}|_{CR^{min}|\phi_u=1,\phi_y=0.05,\phi_x=0} > SAC^{LOLR}|_{CR^{min}|\phi_u=1,\phi_x=\phi_y=0} > SAC^{LOLR}|_{CR^{min}=0}$ ). This result is illustrated in Figure 5 by the broken line.

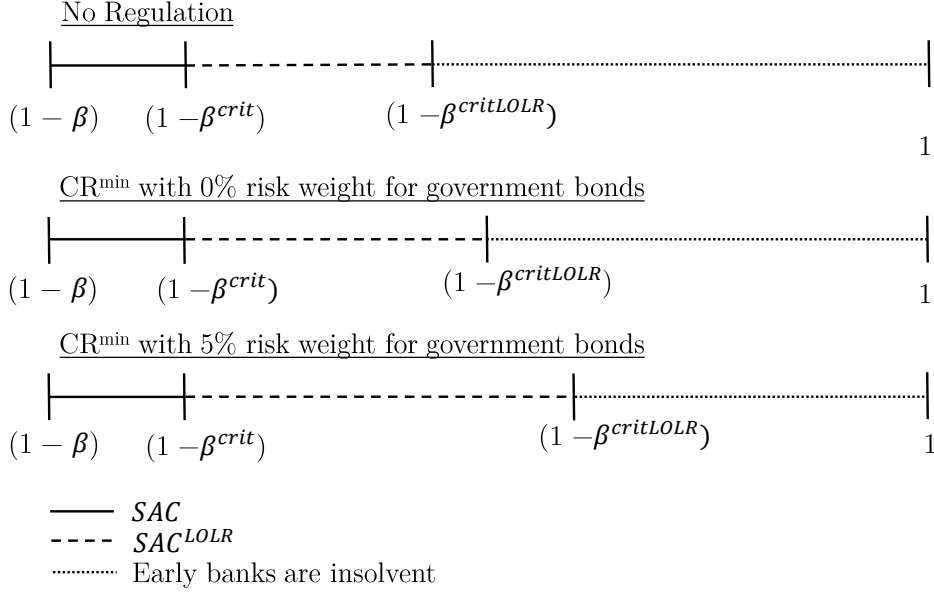


Figure 5: Government Bond Shock-Absorbing Capacity of the Banking Sector

## 7 Summary

In many countries within the EU, banks hold large undiversified amounts of government bonds in their portfolios. These bank sovereign exposures can act as a significant financial contagion channel between sovereigns and banks. The European sovereign debt crisis of 2010 onwards highlighted that some countries within the EU were having severe problems with repaying or refinancing their debt. The resulting price drops of sovereign bonds severely strained the banks' balance sheets. Against this background, there is an ongoing debate about whether the abolishment of the preferential treatment of government bonds in bank capital regulation (sovereign debt of EU member states are still considered as risk-free and thus do not have to be backed with equity capital) can weaken this finan-

cial contagion channel between sovereigns and banks. Our paper adds to this debate in two ways. First, by analysing the consequences of introducing capital requirements for sovereign bonds, for bank investment and financing behaviour. Second, by investigating how much these additional capital requirements thus contribute to making the banking sector more resilient against sovereign debt crises.

As pointed out, for example, by Gennaioli et al. (2014a) an important reason for relatively large government bond holdings is that banks use them to manage their everyday business. Capturing this idea, in the model presented in this paper, banks hold government bonds to balance their idiosyncratic liquidity needs by using an interbank market for government bonds. Increasing sovereign solvency doubts may induce a price drop for government bonds, implying liquidity issues in the banking sector leading to bank insolvencies as illiquid banks have no opportunity to obtain additional liquidity. Government bond holdings thus create a financial contagion channel. Our model shows that in the absence of a LOLR the introduction of capital requirements in general and for government bonds in particular are not able to weaken this financial contagion channel. However, this will be the case if there is a LOLR. The reason is that banks can obtain additional liquidity from the LOLR against adequate collateral. Per se illiquid loans serve as adequate collateral, and the introduction of capital requirements for government bonds induces banks to increase their investment in these loans. This means that they will be able to get more additional liquidity in case of financial contagion.

Our model shows that on the one hand the introduction of capital requirements also for government bonds leads to a decrease of depositors' expected utility as binding capital requirements restrict the possibility of a beneficial liquidity risk transfer from depositors to investors. However, on the other hand these additional capital requirements will contribute to a more resilient banking sector in case of a sovereign debt crisis conditioned on the existence of a LOLR. In this context, it should be noted that our paper does not allow for a comprehensive welfare analysis of introducing capital requirements for sovereign bond – and it was also not the aim of the paper.

## A Appendix A

*Proof I.* Using the Lagrangian  $\mathcal{L}$ , the bank's optimisation problem can be formulated as

$$\begin{aligned} \max_{x,y,u} \mathcal{L} = & 0.5\ln(c_1) + 0.5[0.93 \cdot 0.98\ln(c_{2Hh}) + 0.93 \cdot 0.02\ln(c_{2Hl}) \\ & + 0.07 \cdot 0.98\ln(c_{2Lh}) + 0.07 \cdot 0.02\ln(c_{2Ll})] \\ & - \lambda(x + y + u - 1) - \mu_x x - \mu_y y - \mu_u u, \end{aligned} \quad (\text{A.1})$$

$$\begin{aligned} \text{with } c_1 &= x + yp^{**}, \\ c_{2Hh} &= 1.54u + \left(\frac{x}{p^{**}} + y\right) 1.3, \\ c_{2Hl} &= 1.54u + \left(\frac{x}{p^{**}} + y\right) 0.3, \\ c_{2Lh} &= 0.25u + \left(\frac{x}{p^{**}} + y\right) 1.3, \\ c_{2Ll} &= 0.25u + \left(\frac{x}{p^{**}} + y\right) 0.3, \end{aligned}$$

where  $\lambda$  is the Lagrange multiplier corresponding to the budget constraint (11) and  $\mu_x, \mu_y$  and  $\mu_u$  are the Lagrange multipliers corresponding to the non-negativity conditions (12). Considering that  $p^{**} = 1$  (see Section 4) banks equally split their investment in liquid assets in order to hedge their idiosyncratic liquidity risks, so we have  $x^* = y^* = 0.5z^*$  (for a detailed explanation see Section 5.1). Differentiating  $\mathcal{L}$  with respect to  $z, u, \lambda, \mu_z$  and  $\mu_u$  gives

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial z} = & \frac{0.5}{z} + \frac{0.5 \cdot 0.93 \cdot 0.98 \cdot 1.3}{1.3z + 1.54u} + \frac{0.5 \cdot 0.93 \cdot 0.02 \cdot 0.3}{0.3z + 1.54u} \\ & + \frac{0.5 \cdot 0.07 \cdot 0.98 \cdot 1.3}{1.3z + 0.25u} + \frac{0.5 \cdot 0.07 \cdot 0.02 \cdot 0.3}{0.3z + 0.25u} - \lambda - \mu_z \stackrel{!}{=} 0, \end{aligned} \quad (\text{A.2})$$

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial u} = & \frac{0.5 \cdot 0.93 \cdot 0.98 \cdot 1.54}{1.3z + 1.54u} + \frac{0.5 \cdot 0.93 \cdot 0.02 \cdot 1.54}{0.3z + 1.54u} \\ & + \frac{0.5 \cdot 0.07 \cdot 0.98 \cdot 0.25}{1.3z + 0.25u} + \frac{0.5 \cdot 0.07 \cdot 0.02 \cdot 0.25}{0.3z + 0.25u} - \lambda - \mu_u \stackrel{!}{=} 0, \end{aligned} \quad (\text{A.3})$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = 1 - z - u \stackrel{!}{=} 0, \quad (\text{A.4})$$

$$\frac{\partial \mathcal{L}}{\partial \mu_z} = -z \stackrel{!}{=} 0, \quad (\text{A.5})$$

$$\frac{\partial \mathcal{L}}{\partial \mu_u} = -u \stackrel{!}{=} 0. \quad (\text{A.6})$$

Multiplying both sides of equation (A.2) with  $z$ , of equation (A.3) with  $u$ , adding the two equations and regarding equation (A.4), we obtain  $\lambda = 1$ . Testing whether a non-negativity constraint binds, reveals that this constraint binds for  $u$ , so that  $u^* = 0$  and hence  $\mu_u \neq 0$ . Considering the constraint (11) and  $u^* = 0$ , induces that  $z^* = 1$ , i.e. the representative bank invests its total amount of deposits in liquid assets. ■

*Proof II.* When equity capital is available for banks their optimisation problem reads

$$\begin{aligned} \max_{x,y,u,e_{2Hh},e_{2Hl},e_{2Lh},e_{2Ll}} \quad & \mathcal{L} = 0.5\ln(c_1) + 0.5[0.93 \cdot 0.98\ln(c_{2Hh}) + 0.93 \cdot 0.02\ln(c_{2Hl}) \\ & + 0.07 \cdot 0.98\ln(c_{2Lh}) + 0.07 \cdot 0.02\ln(c_{2Ll})] - \lambda \left( x + y + u \right. \\ & \left. - 1 - \left[ \frac{0.5}{1.5}(1.4497u + 0.9114e_{2Hh} + 0.0186e_{2Hl} \right. \right. \\ & \left. \left. + 0.0686e_{2Lh} + 0.0014e_{2Ll}) \right] \right) - \mu_x x - \mu_y y - \mu_u u \\ & - \mu_{e_{2Hh}} e_{2Hh} - \mu_{e_{2Hl}} e_{2Hl} - \mu_{e_{2Lh}} e_{2Lh} - \mu_{e_{2Ll}} e_{2Ll}, \end{aligned} \quad (\text{A.7})$$

$$\begin{aligned} \text{with} \quad & c_1 = x + yp^{**}, \\ & c_{2Hh} = 1.54u + \left( \frac{x}{p^{**}} + y \right) 1.3 - e_{2Hh}, \\ & c_{2Hl} = 1.54u + \left( \frac{x}{p^{**}} + y \right) 0.3 - e_{2Hl}, \\ & c_{2Lh} = 0.25u + \left( \frac{x}{p^{**}} + y \right) 1.3 - e_{2Lh}, \\ & c_{2Ll} = 0.25u + \left( \frac{x}{p^{**}} + y \right) 0.3 - e_{2Ll}, \end{aligned}$$

where  $\lambda$  is the Lagrange multiplier corresponding to the budget constraint (11). We capture the investors' incentive-compatibility constraint (9) by respectively replacing  $e_0$  in the budget constraint. The variables  $\mu_x, \mu_y, \mu_u, \mu_{e_{2Hh}}, \mu_{e_{2Hl}}, \mu_{e_{2Lh}}, \mu_{e_{2Ll}}$  are the Lagrange multipliers corresponding to the non-negativity conditions (12). As the same argumentation holds as in sections 4 and 5.1, we have  $p^{**} = 1$  and  $x^* = y^* = 0.5z^*$ . By differentiating

the Lagrange function with respect to  $z, u, e_{2Hh}, e_{2Hl}, e_{2Lh}, e_{2Ll}, \mu_z, \mu_u, \lambda, \mu_{e_{2Hh}}, \mu_{e_{2Hl}}, \mu_{e_{2Lh}}$  and  $\mu_{e_{2Ll}}$ , we obtain

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial z} = & \frac{0.5}{z} + \frac{0.5 \cdot 0.93 \cdot 0.98 \cdot 1.3}{1.3z + 1.54u - e_{2Hh}} + \frac{0.5 \cdot 0.93 \cdot 0.02 \cdot 0.3}{0.3z + 1.54u - e_{2Hl}} \\ & + \frac{0.5 \cdot 0.07 \cdot 0.98 \cdot 1.3}{1.3z + 0.25u - e_{2Lh}} + \frac{0.5 \cdot 0.07 \cdot 0.02 \cdot 0.3}{0.3z + 0.25u - e_{2Ll}} - \lambda - \mu_z \stackrel{!}{=} 0, \end{aligned} \quad (\text{A.8})$$

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial u} = & \frac{0.5 \cdot 0.93 \cdot 0.98 \cdot 1.54}{1.3z + 1.54u - e_{2Hh}} + \frac{0.5 \cdot 0.93 \cdot 0.02 \cdot 1.54}{0.3z + 1.54u - e_{2Hl}} \\ & + \frac{0.5 \cdot 0.07 \cdot 0.98 \cdot 0.25}{1.3z + 0.25u - e_{2Lh}} + \frac{0.5 \cdot 0.07 \cdot 0.02 \cdot 0.25}{0.3z + 0.25u - e_{2Ll}} \\ & - \lambda \left( 1 - \left( \frac{0.5 \cdot 1.4497}{1.5} \right) \right) - \mu_u \stackrel{!}{=} 0, \end{aligned} \quad (\text{A.9})$$

$$\frac{\partial \mathcal{L}}{\partial e_{2Hh}} = \frac{0.5 \cdot 0.93 \cdot 0.98 \cdot (-1)}{1.3z + 1.54u - e_{2Hh}} - \lambda \left( -\frac{0.5}{1.5} \cdot 0.9114 \right) - \mu_{e_{2Hh}} \stackrel{!}{=} 0, \quad (\text{A.10})$$

$$\frac{\partial \mathcal{L}}{\partial e_{2Hl}} = \frac{0.5 \cdot 0.93 \cdot 0.02 \cdot (-1)}{0.3z + 1.54u - e_{2Hl}} - \lambda \left( -\frac{0.5}{1.5} \cdot 0.0186 \right) - \mu_{e_{2Hl}} \stackrel{!}{=} 0, \quad (\text{A.11})$$

$$\frac{\partial \mathcal{L}}{\partial e_{2Lh}} = \frac{0.5 \cdot 0.07 \cdot 0.98 \cdot (-1)}{1.3z + 0.25u - e_{2Lh}} - \lambda \left( -\frac{0.5}{1.5} \cdot 0.0686 \right) - \mu_{e_{2Lh}} \stackrel{!}{=} 0, \quad (\text{A.12})$$

$$\frac{\partial \mathcal{L}}{\partial e_{2Ll}} = \frac{0.5 \cdot 0.07 \cdot 0.02 \cdot (-1)}{0.3z + 0.25u - e_{2Ll}} - \lambda \left( -\frac{0.5}{1.5} \cdot 0.0014 \right) - \mu_{e_{2Ll}} \stackrel{!}{=} 0, \quad (\text{A.13})$$

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial \lambda} = & z + u - 1 - \left[ \frac{0.5}{1.5} (1.4497u + 0.9114e_{2Hh} + 0.0186e_{2Hl} \right. \\ & \left. + 0.0686e_{2Lh} + 0.0014e_{2Ll}) \right] \stackrel{!}{=} 0, \end{aligned} \quad (\text{A.14})$$

$$\frac{\partial \mathcal{L}}{\partial \mu_z} = -z \stackrel{!}{=} 0, \quad (\text{A.15})$$

$$\frac{\partial \mathcal{L}}{\partial \mu_u} = -u \stackrel{!}{=} 0, \quad (\text{A.16})$$

$$\frac{\partial \mathcal{L}}{\partial \mu_{e_{2Hh}}} = -e_{2Hh} \stackrel{!}{=} 0, \quad (\text{A.17})$$

$$\frac{\partial \mathcal{L}}{\partial \mu_{e_{2Hl}}} = -e_{2Hl} \stackrel{!}{=} 0, \quad (\text{A.18})$$

$$\frac{\partial \mathcal{L}}{\partial \mu_{e_{2Lh}}} = -e_{2Lh} \stackrel{!}{=} 0, \quad (\text{A.19})$$

$$\frac{\partial \mathcal{L}}{\partial \mu_{e_{2Ll}}} = -e_{2Ll} \stackrel{!}{=} 0. \quad (\text{A.20})$$

Multiplying both sides of equation (A.8) with  $z$ , of (A.9) with  $u$ , of (A.10) with  $e_{2Hh}$ , of (A.11) with  $e_{2Hl}$ , of (A.12) with  $e_{2Lh}$  and of (A.13) with  $e_{2Ll}$ , adding the six equations and regarding equation (A.14), we again obtain  $\lambda = 1$ . After testing which non-negativity

conditions bind, we derive that the non-negativity conditions for  $e_{Hh}, e_{Hl}, e_{Ll}$  and  $e_{Ll}$  become binding, i.e.  $e_{Hh}^* = e_{Hl}^* = e_{Ll}^* = e_{Ll}^* = 0$  and thus  $\mu_{e_{2Hh}} = \mu_{e_{2Hl}} = \mu_{e_{2Lh}} = \mu_{e_{2Ll}} \neq 0$ . Solving then for  $z^*$  and  $u^*$  we get  $z^* = 0.9088$  and  $u^* = 0.1765$  and regarding the constraint (9) the optimal amount of equity capital is  $e_0^* = 0.0853$ . ■

*Proof III.* When a bank faces capital requirements for loans ( $CR^{min} = 0.5799 = \frac{e_0}{u}$ ), its optimisation problem can be formulated as

$$\begin{aligned}
\max_{x,y,u,e_{2Hh},e_{2Hl},e_{2Lh},e_{2Ll}} \mathcal{L} = & 0.5\ln(c_1) + 0.5[0.93 \cdot 0.98\ln(c_{2Hh}) + 0.93 \cdot 0.02\ln(c_{2Hl}) \\
& + 0.07 \cdot 0.98\ln(c_{2Lh}) + 0.07 \cdot 0.02\ln(c_{2Ll})] \\
& - \lambda \left( x + y + u - 1 - \left[ \frac{0.5}{1.5} (1.4497u + 0.9114e_{2Hh} \right. \right. \\
& \left. \left. + 0.0186e_{2Hl} + 0.0686e_{2Lh} + 0.0014e_{2Ll}) \right] \right) \quad (A.21) \\
& - \mu_x x - \mu_y y - \mu_u u - \mu_{e_{2Hh}} e_{2Hh} \\
& - \mu_{e_{2Hl}} e_{2Hl} - \mu_{e_{2Lh}} e_{2Lh} - \mu_{e_{2Ll}} e_{2Ll} - \mu_{CR} \\
& \left( \frac{0.5}{\frac{1.5}{u}} \left[ 1.4497u + 0.9114e_{2Hh} + 0.0186e_{2Hl} \right. \right. \\
& \left. \left. + 0.0686e_{2Lh} + 0.0014e_{2Ll} \right] - 0.5799 \right),
\end{aligned}$$

$$\text{with } c_1 = x + yp^{**},$$

$$c_{2Hh} = 1.54u + \left( \frac{x}{p^{**}} + y \right) 1.3 - e_{2Hh},$$

$$c_{2Hl} = 1.54u + \left( \frac{x}{p^{**}} + y \right) 0.3 - e_{2Hl},$$

$$c_{2Lh} = 0.25u + \left( \frac{x}{p^{**}} + y \right) 1.3 - e_{2Lh},$$

$$c_{2Ll} = 0.25u + \left( \frac{x}{p^{**}} + y \right) 0.3 - e_{2Ll},$$

where  $\lambda$  is the Lagrange multiplier corresponding to the budget constraint (11). We capture the investors' incentive-compatibility constraint (9) by respectively replacing  $e_0$  in the budget constraint and in the regulation constraint. The variables  $\mu_x, \mu_y, \mu_u, \mu_{e_{2Hh}}, \mu_{e_{2Hl}}, \mu_{e_{2Lh}}$  and  $\mu_{e_{2Ll}}$  are the Lagrange multipliers corresponding to the non-negativity conditions (12) and  $\mu_{CR}$  is the Lagrange multiplier corresponding

to the regulation constraint (10). Considering that  $p^{**} = 1$  (see Section 4) as well as  $x^* = y^* = 0.5z^*$  (for a detailed explanation see Section 5.1) and differentiating  $\mathcal{L}$  with respect to  $z, u, e_{2Hh}, e_{2Hl}, e_{2Lh}, e_{2Ll}, \lambda, \mu_{CR}, \mu_{e_{2Hh}}, \mu_{e_{2Hl}}, \mu_{e_{2Lh}}$  and  $\mu_{e_{2Ll}}$  gives

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial z} = & \frac{0.5}{z} + \frac{0.5 \cdot 0.93 \cdot 0.98 \cdot 1.3}{1.3z + 1.54u - e_{2Hh}} + \frac{0.5 \cdot 0.93 \cdot 0.02 \cdot 0.3}{0.3z + 1.54u - e_{2Hl}} \\ & + \frac{0.5 \cdot 0.07 \cdot 0.98 \cdot 1.3}{1.3z + 0.25u - e_{2Lh}} + \frac{0.5 \cdot 0.07 \cdot 0.02 \cdot 0.3}{0.3z + 0.25u - e_{2Ll}} - \lambda - \mu_z \stackrel{!}{=} 0, \end{aligned} \quad (\text{A.22})$$

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial u} = & \frac{0.5 \cdot 0.93 \cdot 0.98 \cdot 1.54}{1.3z + 1.54u - e_{2Hh}} + \frac{0.5 \cdot 0.93 \cdot 0.02 \cdot 1.54}{0.3z + 1.54u - e_{2Hl}} \\ & + \frac{0.5 \cdot 0.07 \cdot 0.98 \cdot 0.25}{1.3z + 0.25u - e_{2Lh}} + \frac{0.5 \cdot 0.07 \cdot 0.02 \cdot 0.25}{0.3z + 0.25u - e_{2Ll}} \\ & - \lambda \left( 1 - \left( \frac{0.5 \cdot 1.4497}{1.5} \right) \right) + \mu_{CR} \left( \frac{0.5}{u^2} [0.9114e_{2Hh} \right. \\ & \left. + 0.0186e_{2Hl} + 0.0686e_{2Lh} + 0.0014e_{2Ll}] \right) - \mu_u \stackrel{!}{=} 0, \end{aligned} \quad (\text{A.23})$$

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial e_{2Hh}} = & \frac{0.5 \cdot 0.93 \cdot 0.98 \cdot (-1)}{1.3z + 1.54u - e_{2Hh}} - \lambda \left( -\frac{0.5}{1.5} \cdot 0.9114 \right) - \mu_{e_{2Hh}} \\ & - \mu_{CR} \left( \frac{0.5}{u} \cdot 0.9114 \right) \stackrel{!}{=} 0, \end{aligned} \quad (\text{A.24})$$

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial e_{2Hl}} = & \frac{0.5 \cdot 0.93 \cdot 0.02 \cdot (-1)}{0.3z + 1.54u - e_{2Hl}} - \lambda \left( -\frac{0.5}{1.5} \cdot 0.0186 \right) - \mu_{e_{2Hl}} \\ & - \mu_{CR} \left( \frac{0.5}{u} \cdot 0.0186 \right) \stackrel{!}{=} 0, \end{aligned} \quad (\text{A.25})$$

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial e_{2Lh}} = & \frac{0.5 \cdot 0.07 \cdot 0.98 \cdot (-1)}{1.3z + 0.25u - e_{2Lh}} - \lambda \left( -\frac{0.5}{1.5} \cdot 0.0686 \right) - \mu_{e_{2Lh}} \\ & - \mu_{CR} \left( \frac{0.5}{u} \cdot 0.0686 \right) \stackrel{!}{=} 0, \end{aligned} \quad (\text{A.26})$$

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial e_{2Ll}} = & \frac{0.5 \cdot 0.07 \cdot 0.02 \cdot (-1)}{0.3z + 0.25u - e_{2Ll}} - \lambda \left( -\frac{0.5}{1.5} \cdot 0.0014 \right) - \mu_{e_{2Ll}} \\ & - \mu_{CR} \left( \frac{0.5}{u} \cdot 0.0014 \right) \stackrel{!}{=} 0, \end{aligned} \quad (\text{A.27})$$

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial \lambda} = & z + u - 1 - \left[ \frac{0.5}{1.5} (1.4497u + 0.9114e_{2Hh} + 0.0186e_{2Hl} \right. \\ & \left. + 0.0686e_{2Lh} + 0.0014e_{2Ll}) \right] \stackrel{!}{=} 0. \end{aligned} \quad (\text{A.28})$$

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial \mu_{CR}} = & \frac{0.5}{u} \left[ 1.4497u + 0.9114e_{2Hh} + 0.0186e_{2Hl} \right. \\ & \left. + 0.0686e_{2Lh} + 0.0014e_{2Ll} \right] - 0.5799 \stackrel{!}{=} 0. \end{aligned} \quad (\text{A.29})$$



$$\frac{\partial \mathcal{L}}{\partial \mu_z} = -z \stackrel{!}{=} 0, \quad (\text{A.30})$$

$$\frac{\partial \mathcal{L}}{\partial \mu_u} = -u \stackrel{!}{=} 0, \quad (\text{A.31})$$

$$\frac{\partial \mathcal{L}}{\partial \mu_{e_{2Hh}}} = -e_{2Hh} \stackrel{!}{=} 0, \quad (\text{A.32})$$

$$\frac{\partial \mathcal{L}}{\partial \mu_{e_{2Hl}}} = -e_{2Hl} \stackrel{!}{=} 0, \quad (\text{A.33})$$

$$\frac{\partial \mathcal{L}}{\partial \mu_{e_{2Lh}}} = -e_{2Lh} \stackrel{!}{=} 0, \quad (\text{A.34})$$

$$\frac{\partial \mathcal{L}}{\partial \mu_{e_{2Ll}}} = -e_{2Ll} \stackrel{!}{=} 0. \quad (\text{A.35})$$

Multiplying both sides of equation (A.22) with  $z$ , of (A.23) with  $u$ , of (A.24) with  $e_{2Hh}$ , of (A.25) with  $e_{2Hl}$ , of (A.26) with  $e_{2Lh}$  and of (A.27) with  $e_{2Ll}$ , adding the six equations and regarding equation (A.28), we again obtain  $\lambda = 1$ . After testing which non-negativity conditions bind, we derive that the non-negativity conditions for  $e_{Hl}, e_{Ll}$  and  $e_{Ll}$  become binding, i.e.  $e_{Hl}^* = e_{Ll}^* = e_{Ll}^* = 0$  and thus  $\mu_{e_{2Hl}} = \mu_{e_{2Lh}} = \mu_{e_{2Ll}} \neq 0$ . Solving then for  $z^*$ ,  $u^*$  and  $e_{2Hh}^*$  we get  $z^* = 0.9162$ ,  $u^* = 0.1995$  and  $e_{2Hh}^* = 0.0635$ . Regarding constraint (9), the optimal amount of equity capital is  $e_0^* = 0.1157$ . ■

*Proof IV.* When banks face capital requirements for loans and government bonds ( $CR^{min} = 0.5799 = \frac{e_0}{u+0.05y}$ ), their optimisation problem becomes

$$\begin{aligned} \max_{x,y,u,e_{2Hh},e_{2Hl},e_{2Lh},e_{2Ll}} \mathcal{L} = & 0.5 \ln(c_1) + 0.5 [0.93 \cdot 0.98 \ln(c_{2Hh}) + 0.93 \cdot 0.02 \ln(c_{2Hl}) \\ & + 0.07 \cdot 0.98 \ln(c_{2Lh}) + 0.07 \cdot 0.02 \ln(c_{2Ll})] \\ & - \lambda \left( x + y + u - 1 - \left[ \frac{0.5}{1.5} (1.4497u + 0.9114e_{2Hh} \right. \right. \\ & \left. \left. + 0.0186e_{2Hl} + 0.0686e_{2Lh} + 0.0014e_{2Ll}) \right] \right) \\ & - \mu_x x - \mu_y y - \mu_u u - \mu_{e_{2Hh}} e_{2Hh} \\ & - \mu_{e_{2Hl}} e_{2Hl} - \mu_{e_{2Lh}} e_{2Lh} - \mu_{e_{2Ll}} e_{2Ll} - \mu_{CR} \\ & \left( \frac{0.5}{u+0.05y} \left[ 1.4497u + 0.9114e_{2Hh} + 0.0186e_{2Hl} \right. \right. \\ & \left. \left. + 0.0686e_{2Lh} + 0.0014e_{2Ll} \right] - 0.5799 \right), \end{aligned} \quad (\text{A.36})$$

with  $c_1 = x + yp^{**}$ ,

$$\begin{aligned} c_{2Hh} &= 1.54u + \left( \frac{x}{p^{**}} + y \right) 1.3 - e_{2Hh}, \\ c_{2Hl} &= 1.54u + \left( \frac{x}{p^{**}} + y \right) 0.3 - e_{2Hl}, \\ c_{2Lh} &= 0.25u + \left( \frac{x}{p^{**}} + y \right) 1.3 - e_{2Lh}, \\ c_{2Ll} &= 0.25u + \left( \frac{x}{p^{**}} + y \right) 0.3 - e_{2Ll}, \end{aligned}$$

where  $\lambda$  is the Lagrange multiplier corresponding to the budget constraint (11). We capture the investors' incentive-compatibility constraint (9) by respectively replacing  $e_0$  in the budget constraint and in the regulation constraint. The variables  $\mu_x, \mu_y, \mu_u, \mu_{e_{2Hh}}, \mu_{e_{2Hl}}, \mu_{e_{2Lh}}, \mu_{e_{2Ll}}$  are the Lagrange multipliers corresponding to the non-negativity conditions (12) and  $\mu_{CR}$  is the Lagrange multiplier corresponding to the regulation constraint (10). Considering that  $p^{**} = 1$  (see Section 4) banks equally split their investment in liquid assets ( $x^* = y^* = 0.5z^*$ ) also when sovereign bonds are subject to capital regulation. By differentiating  $\mathcal{L}$  with respect to  $z, u, e_{2Hh}, e_{2Hl}, e_{2Lh}, e_{2Ll}, \lambda, \mu_{CR}, \mu_{e_{2Hh}}, \mu_{e_{2Hl}}, \mu_{e_{2Lh}}$  and  $\mu_{e_{2Ll}}$  we obtain

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial z} &= \frac{0.5}{z} + \frac{0.5 \cdot 0.93 \cdot 0.98 \cdot 1.3}{1.3z + 1.54u - e_{2Hh}} + \frac{0.5 \cdot 0.93 \cdot 0.02 \cdot 0.3}{0.3z + 1.54u - e_{2Hl}} \\ &\quad + \frac{0.5 \cdot 0.07 \cdot 0.98 \cdot 1.3}{1.3z + 0.25u - e_{2Lh}} + \frac{0.5 \cdot 0.07 \cdot 0.02 \cdot 0.3}{0.3z + 0.25u - e_{2Ll}} - \lambda - \mu_z \\ &\quad + \mu_{CR} \left( \frac{\frac{0.5 \cdot 0.025}{1.5}}{(u + 0.025z)^2} [1.4497u + 0.9114e_{2Hh} \right. \\ &\quad \left. + 0.0186e_{2Hl} + 0.0686e_{2Lh} + 0.0014e_{2Ll}] \right) \stackrel{!}{=} 0, \end{aligned} \tag{A.37}$$

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial u} &= \frac{0.5 \cdot 0.93 \cdot 0.98 \cdot 1.54}{1.3z + 1.54u - e_{2Hh}} + \frac{0.5 \cdot 0.93 \cdot 0.02 \cdot 1.54}{0.3z + 1.54u - e_{2Hl}} \\ &\quad + \frac{0.5 \cdot 0.07 \cdot 0.98 \cdot 0.25}{1.3z + 0.25u - e_{2Lh}} + \frac{0.5 \cdot 0.07 \cdot 0.02 \cdot 0.25}{0.3z + 0.25u - e_{2Ll}} \\ &\quad - \lambda \left( 1 - \left( \frac{0.5 \cdot 1.4497}{1.5} \right) \right) + \mu_{CR} \left( \frac{\frac{0.5}{1.5}}{(u + 0.025z)^2} \right. \\ &\quad \left. [1.4497 \cdot 0.025z + 0.9114e_{2Hh} + 0.0186e_{2Hl} \right. \\ &\quad \left. + 0.0686e_{2Lh} + 0.0014e_{2Ll}] \right) - \mu_u \stackrel{!}{=} 0, \end{aligned} \tag{A.38}$$

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial e_{2Hh}} &= \frac{0.5 \cdot 0.93 \cdot 0.98 \cdot (-1)}{1.3z + 1.54u - e_{2Hh}} - \lambda \left( -\frac{0.5}{1.5} \cdot 0.9114 \right) - \mu_{e_{2Hh}} \\ &\quad - \mu_{CR} \left( \frac{0.3038}{u + 0.025z} \right) \stackrel{!}{=} 0,\end{aligned}\tag{A.39}$$

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial e_{2Hl}} &= \frac{0.5 \cdot 0.93 \cdot 0.02 \cdot (-1)}{0.3z + 1.54u - e_{2Hl}} - \lambda \left( -\frac{0.5}{1.5} \cdot 0.0186 \right) - \mu_{e_{2Hl}} \\ &\quad - \mu_{CR} \left( \frac{0.062}{u + 0.025z} \right) \stackrel{!}{=} 0,\end{aligned}\tag{A.40}$$

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial e_{2Lh}} &= \frac{0.5 \cdot 0.07 \cdot 0.98 \cdot (-1)}{1.3z + 0.25u - e_{2Lh}} - \lambda \left( -\frac{0.5}{1.5} \cdot 0.0686 \right) - \mu_{e_{2Lh}} \\ &\quad - \mu_{CR} \left( \frac{0.02286}{u + 0.025z} \right) \stackrel{!}{=} 0,\end{aligned}\tag{A.41}$$

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial e_{2Ll}} &= \frac{0.5 \cdot 0.07 \cdot 0.02 \cdot (-1)}{0.3z + 0.25u - e_{2Ll}} - \lambda \left( -\frac{0.5}{1.5} \cdot 0.0014 \right) - \mu_{e_{2Ll}} \\ &\quad - \mu_{CR} \left( \frac{0.00046}{u + 0.025z} \right) \stackrel{!}{=} 0,\end{aligned}\tag{A.42}$$

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial \lambda} &= z + u - 1 - \left[ \frac{0.5}{1.5} (1.4497u + 0.9114e_{2Hh} + 0.0186e_{2Hl} \right. \\ &\quad \left. + 0.0686e_{2Lh} + 0.0014e_{2Ll}) \right] \stackrel{!}{=} 0.\end{aligned}\tag{A.43}$$

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial \mu_{CR}} &= \frac{\frac{0.5}{1.5}}{u + 0.025z} \left[ 1.4497u + 0.9114e_{2Hh} + 0.0186e_{2Hl} \right. \\ &\quad \left. + 0.0686e_{2Lh} + 0.0014e_{2Ll} \right] - 0.5799 \stackrel{!}{=} 0.\end{aligned}\tag{A.44}$$

$$\frac{\partial \mathcal{L}}{\partial \mu_z} = -z \stackrel{!}{=} 0,\tag{A.45}$$

$$\frac{\partial \mathcal{L}}{\partial \mu_u} = -u \stackrel{!}{=} 0,\tag{A.46}$$

$$\frac{\partial \mathcal{L}}{\partial \mu_{e_{2Hh}}} = -e_{2Hh} \stackrel{!}{=} 0,\tag{A.47}$$

$$\frac{\partial \mathcal{L}}{\partial \mu_{e_{2Hl}}} = -e_{2Hl} \stackrel{!}{=} 0,\tag{A.48}$$

$$\frac{\partial \mathcal{L}}{\partial \mu_{e_{2Lh}}} = -e_{2Lh} \stackrel{!}{=} 0,\tag{A.49}$$

$$\frac{\partial \mathcal{L}}{\partial \mu_{e_{2Ll}}} = -e_{2Ll} \stackrel{!}{=} 0,\tag{A.50}$$

Multiplying both sides of equation (A.37) with  $z$ , of (A.38) with  $u$ , of (A.39) with  $e_{2Hh}$ , of (A.40) with  $e_{2Hl}$ , of (A.41) with  $e_{2Lh}$  and of (A.42) with  $e_{2Ll}$ , adding the 6 equations and regarding equation (A.43), we again obtain  $\lambda = 1$ . After testing which non-negativity conditions bind, we derive that the non-negativity conditions for  $e_{Hl}$ ,  $e_{Ll}$  and  $e_{Ll}$  become binding, i.e.  $e_{Hl}^* = e_{Ll}^* = e_{Ll}^* = 0$  and thus  $\mu_{e_{2Hl}} = \mu_{e_{2Lh}} = \mu_{e_{2Ll}} \neq 0$ . Solving then for

$z^*$ ,  $u^*$  and  $e_{2Hh}^*$ , we get  $z^* = 0.9172$ ,  $u^* = 0.2287$  and  $e_{2Hh}^* = 0.1166$ . Inserting  $z^*$ ,  $u^*$  and  $e_{2Hh}^*$  in constraint (9), the optimal amount of equity capital a bank raises is  $e_0^* = 0.1459$ .

■

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