Bank Liquidity Management, Collateral Quality and Policies[*]

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Abstract

We develop a model in which financial intermediaries hold near-cash assets to protect themselves from shocks. Depending on parameter values, banks may choose to hold too much or too little cash on aggregate compared to the socially optimal amount. The model, therefore, provides a unified framework for thinking, on the one hand, about policy measures that can reduce hoarding of cash by banks and, on the other hand, about liquidity requirements of the type imposed by the new Basel III regulation. Key to our results is a market in which banks can obtain cash by selling or repo-ing their marketable securities. The quantity of cash obtained on this market is determined endogenously by the market value of the marketable securities and is subject to cash-in-the-market pricing. When uncertainty about the value of marketable securities is low, banks hold too little cash and liquidity regulation can achieve a better allocation. When uncertainly about the value of marketable securities is sufficiently high, banks hoard cash leading to a market freeze.

Keywords: money market, liquidity regulation, non conventional monetary policy, cash-in-the-market-pricing

JEL: E58, G28, G21

1. Introduction

This paper studies whether the amount of liquidity (near-cash assets) chosen by financial intermediaries to protect themselves from shocks is optimal from the perspective of social welfare. Liquidity has been an important policy issue since the 2007-2009 financial crisis. The new Basel III regulation introduced liquidity requirements, in the form of the liquidity coverage ratio (LCR) and the net stable funding ratio (NSFR). This type of regulation stems from the concern that financial intermediaries may not hold enough liquid assets. On the other hand, concerns have been expressed that some financial intermediaries may be hoarding liquid assets such as cash, leading to market freezes, particularly in Europe during the Subprime and Euro crisis (ECB, 2014).[1] In light of these concerns, it seems important to understand under which circumstances the private decisions of decentralized financial intermediaries may lead to too much or too little liquidity in the financial system. Our model provides a unified framework to think about these issues and the policy interventions that could lead to better outcomes.

We develop a simple model of banks that are subject to a liquidity shock, in the spirit of Diamond and Dybvig (1983) and Holmstrom and Tirole (1998). Our banks are endowed with a non-tradable project, which we can think of as a commercial loan, that may require additional cash to successfully come to maturity. Banks also need to decide how much near-cash assets and marketable securities they want to hold. We think of near-cash assets as level 1 high quality liquid assets, such as central bank reserves and sovereign bonds. Examples of marketable securities in our model would be corporate bonds or equities. Marketable securities have a higher expected return than cash but their return is uncertain. In addition, these securities cannot be used directly to meet the liquidity shock, but they can be exchanged against cash in a market. The market may be interpreted either as a secured money market in which banks lend temporarily their securities against cash (e.g. in a repo transaction) or as a market in which the securities are sold.[2] Banks holding securities that turn out to have a low return may not be able to raise sufficient cash when they are hit by a liquidity shock.

We assume that the non-marketable project cannot be pledged or sold to other banks.

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[1]For example, the Wall Street Journal reported "Europe's biggest banks are continuing to stash more money at central banks, a move that reflects their lingering fears about the financial system despite signs of improvement." (WSJ "Large European Banks Stash Cash" Nov. 13 2012). For academic work on the topic see Acharya and Merrouche (2012) for the UK, Heider et al. (2015) for Euro zone, de Haan and van den End (2013) for the Netherlands. See also Ashcraft et al. (2011) for empirical evidence of precautionary holding of reserves by U.S. banks in 2007-2008.

[2]As is usual in the literature (for example Allen et al. (2009)), there is no substantial difference between a secured lending market and a market for outright purchases in our model. Indeed, since the return on marketable securities is publicly known when the market opens in our model, there is no difference between selling marketable securities and using them as collateral in secured loans.

That project may not be good collateral because its value could depend on the specific skills of the banker so that its value is very low when the bank fails, which is precisely when the collateral is needed (Diamond and Rajan, 2001).[3] In contrast, marketable securities are good collateral because their value does not depend on the survival of the bank that pledged them.

In choosing the optimal amount of cash, a planner weighs the benefit of an additional unit of cash, which can be used to continue non-marketable projects, against the cost, which is the foregone return on tradable assets. At the optimum, the planner chooses to hold exactly enough cash, on aggregate, to continue all non-marketable projects, because we assume that the return on those projects is sufficiently high.

We show that banks may choose to hold too much or too little cash on aggregate, in equilibrium, depending on parameter values, compared to the socially optimal amount. When uncertainly about the value of the marketable asset is low, banks hold too little cash because they do not internalize the effect their cash-holding decision on the equilibrium price of securities. Low cash holding implies that the value of securities is low because of cash-in-the-market pricing. This, in turns, reduces the extent to which banks hit by a liquidity shock can continue their non-marketable project.

If the uncertainty about the value of the marketable securities is sufficiently high, banks choose to hoard cash; that is, banks hold enough cash to continue their long-term project without needing to access the market. This leads to a market "freeze". Nevertheless, we show that this allocation is constrained optimal.

Our work relates to the body of research analyzing liquidity choices by financial institutions, and the link with asset prices. Following Allen and Gale (2004, 2005), a number of papers have analyzed how *cash-in-the-market-pricing* affects banks' liquidity decisions and the possibility of central bank or public intervention. In particular, our paper is related to Allen et al. (2009), Freixas et al. (2011) and Gale and Yorulmazer (2013). An important difference between these papers and ours is that there can be either too much or too little cash in equilibrium in our model. Acharya et al. (2010) shows how the expectation of fire sales can lead to liquidity choices that are excessive from a social viewpoint. The strategic motive for cash hoarding is also analyzed in Acharya et al. (2012) and Gale and Yorulmazer (2013). Other papers show that liquidity hoarding may be a response to an exogenous increase in counterparty risk (Heider et al., 2015). The interplay between endogenous liquidity choices and private information is analyzed in Bolton et al. (2011) and Malherbe (2014). Bolton et al. (2011) focus on a tradeoff between outside and inside liquidity and show that the timing of trade can lead to an equilibrium with too little outside liquidity

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[3]While we do not explicitly model these frictions, a number of informational friction could help explain why the non-marketable project cannot be sold or its return pledged. See for example Gale and Yorulmazer (2013).

and too little cash-in-the-market-pricing. Malherbe (2014) shows how in the absence of cash-in-the-market-pricing initial liquidity choices impose a negative externality on future liquidity by exacerbating adverse selection problems. We consider a setup where information on traded assets is symmetric, and relate socially excessive liquidity to the dispersion of traded assets.

The remainder of the paper proceeds as follows: In section 2 we describe the model. Section 3 characterizes the allocations chosen by the planner, under different constraints. We describe banks' behavior in section 4. The definition and characterization of equilibria is done in section 5. Section 6 considers some policy implications of the model. Section 7 concludes.

2. Environment

The economy is populated by a continuum of identical agents called banks. There are 3 dates indexed by $t \in \{0, 1, 2\}$. At $t = 0$, each bank is endowed with a non-marketable investment project, which we can think of as an industrial loan or a commercial real-estate deal, for example, as well as one unit of cash.[4] Banks can keep the cash or invest in marketable "securities," such as bonds, equities, or asset-backed securities.[5] At $t = 1$, a financial market opens in which each bank can rebalance its portfolio of cash and securities. At $t = 2$, the investment project and the securities mature, and consumption takes place.

2.1. *Cash and securities*

Cash has a net return of 0 and can be costlessly stored from one period to the next. Tradable securities generate a stochastic return at date 2. The actual return depends on the type θ of the security, which is publicly revealed at date 1. That type is H with probability of μ and L otherwise. Tradable securities generate a return R_θ at date 2 with $R_H > R_L$. The expected gross return, denoted $\bar{R}$, is assumed to be greater than 1.

$$\bar{R} \equiv \mathbb{E}\left(R_\theta\right) = \mu R_H + (1 - \mu) R_L > 1 \tag{A1}$$

Let $s \in [0, 1]$ denote the share of the cash in the bank's portfolio at $t = 0$.

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[4]We think of "cash" as including near-cash assets, such as level 1 HQLA.

[5]Empirically we think of this asset as any type of security with an International Security Identification Number (ISIN) or, in the U.S. context, a CUSIP, allowing its trade on an organized market. We do not modeled the process leading to the creation of this type of security.

2.2. *Project*

Each bank holds a project that yields y at $t = 2$. At $t = 1$, with probability λ, the project is hit by a liquidity shock that requires the injection of an extra amount of cash.[6] To simplify notation, we normalize that amount to 1. In Appendix A, we show that our results generalize to the case where the amount of cash to be injected is less than 1. If no cash is injected, the project is liquidated at $t = 1$ and yields nothing. The project is divisible: If a bank does not have enough cash at $t = 1$ to continue the project in its entirety, it can inject $i < 1$ of cash and the project returns $i \cdot y$ at date $t = 2$.

We assume that :

$$y > \bar{R} \tag{A2}$$

This condition implies that, from the perspective of date 0, the return y on the extra amount of cash that needs to be injected to continue to project in case of a liquidity shock is greater than $\bar{R}$, the expected return on securities.

2.3. *Timing*

- $t = 0$: Each bank chooses s, the share of cash it wants to hold, with $1 - s$ representing the share of securities the bank holds.

- $t = 1$: Each bank learns the value of the securities it holds and whether it suffer liquidity shock. The market for securities opens. Bank with a liquidity shock decide whether to inject additional cash to continue their project.

- $t = 2$: The return on long term project and the securities are realized. Banks profit are realized.

3. Market for liquidity

At date 1, a market opens in which banks can trade securities for cash. (equivalently, borrow cash pledging securities) Banks that receive a liquidity shock and need to acquire additional cash to continue their project will seek to sell some or all of their securities holding in this market. Banks that did not receive a liquidity shock, as well as banks that are unwilling to continue their project, may want to purchase securities with their cash.

3.1. *No arbitrage between two segments of market*

There exists two types of securities H, L. Because there is no informational asymmetry about the type of securities between buyers and sellers, the return from purchasing either

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[6]We assume a law of large number holds so that the share of projects affected by the liquidity shock is also λ.

- Banks invest in assets
  - s in cash
  - $1 - s$ in securities

- Liquidity shock realized
  - λ need to inject 1
- Type of security $\{H, L\}$, its value revealed

  $\mu \longrightarrow R_H$

  $1 - \mu \longrightarrow R_L (< R_H)$

- Market for cash/securites trades
  - prices: pR_H, pR_L
- Refinanced projects continued

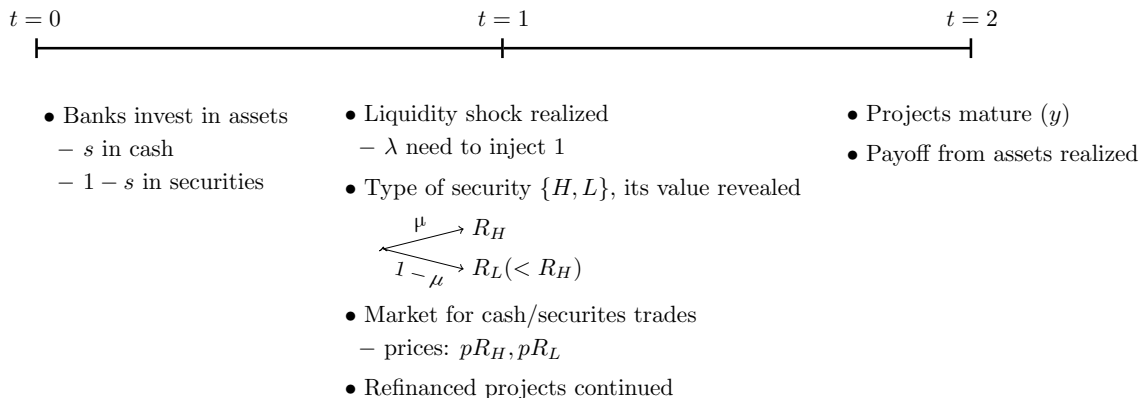- Projects mature (y)
- Payoff from assets realized

Figure 1: Timing

security be same in equilibrium. Let $\frac{1}{p}$ denote the market return on securities. This notation is convenient because the price of security θ can be written as pR_θ. The no-arbitrage condition can thus be expressed as

$$\frac{R_H}{pR_H} = \frac{R_L}{pR_L} = \frac{1}{p}.$$

(1)

3.2. *Demand of cash*

Banks that receive a liquidity shock demand cash and supply securities to the market, in order to continue their nonmarketable project, provided they have an incentive to continue their project. They will do as far as the return of injecting 1 unit of cash in the project or internal return on cash is higher than and its opportunity cost, which is the return on purchasing securities or market return on cash $\frac{1}{p}$. The bank will demand cash if

$$y \geq \frac{1}{p}.$$

(2)

As far as inequality (2) holds, the amount of cash needed by each bank affected by a liquidity shock is $1 - s$. However, the amount of cash effectively demanded by each bank depends on the amount of cash backed by the market value of the securities, pR_θ. On the one hand, if this is higher than 1, the effective demand is not constrained by the market value of the security and as a consequence banks demand $1 - s$. On the other hand, if the price of the securities is lower than 1, the effective demand is constrained by the market value of securities. As a consequence their effective demand only becomes $(1 - s)\, pR_\theta$. If inequality (2) does not hold, banks will not demand any cash.

The individual demand function for cash by a bank holding θ-type security and hit by

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a liquidity shock, denoted $d(p, s, \theta)$, is given by:

$$
d(p, s, \theta) = \begin{cases} (1-s)\min\{1, pR_\theta\} & \frac{1}{p} < y, \\ [0, (1-s)\min\{1, pR_\theta\}] & \text{if} \quad \frac{1}{p} = y, \\ 0 & \frac{1}{p} > y. \end{cases}
$$

The aggregate demand function on cash, D, is given by the sum of the individual demands:

$$
D = \begin{cases} D(s, p) & \frac{1}{p} < y, \\ [0, D(s, p)] & \text{if} \quad \frac{1}{p} = y, \\ 0 & \frac{1}{p} > y. \end{cases} \tag{3}
$$

where

$$
D(s, p) = \lambda(1-s)[\mu\min\{1, pR_H\} + (1-\mu)\min\{1, pR_L\}],
$$

which is equivalent to

$$
D(s, p) = \begin{cases} \lambda(1-s) & \text{if} & \frac{1}{p} \leq R_L, \\ \lambda(1-s)[\mu + (1-\mu)pR_L] & \text{if} \quad R_L < \frac{1}{p} \leq R_H, \\ \lambda(1-s)p\bar{R} & \text{if} & \frac{1}{p} > R_H. \end{cases}
$$

If $\frac{1}{p} \leq R_L$, then banks affected by a liquidity shock can get the cash they need to continue their project regardless of the return on the securities they hold since the market value of the securities they hold is higher than the amount of cash they need. So a fraction λ of banks demand an amount $(1-s)$ of cash. If $R_L < \frac{1}{p} \leq R_H$, then banks affected by a liquidity shock can get as much cash as they would like if they hold securities with return R_H, but are constrained if they hold securities with return R_L. Finally, if $\frac{1}{p} > R_H$, all banks affected by a liquidity shock are constrained and they can demand only the amount of cash backed by the market value of securities they hold. Figure 2 illustrates the aggregate demand function on cash as well as the amount of cash needed in the economy. Upon the value of y and of R_H, demand curve can be one of the two above cases.

3.3. *Supply of cash*

Next, we can derive the supply of cash. Banks may be willing to supply cash to the market, and purchase securities, for two reasons. First, banks that are not subject to the liquidity shock, $(1 - \lambda)$ will be willing to supply cash buying securities if their return is higher than or at least equal to return on cash, $\frac{1}{p} \geq 1$. Second, banks hit by the liquidity shock, λ supply cash and purchase securities if the market return on cash is higher than their internal return on cash *i.e.* return of injecting cash in the project, $\frac{1}{p} \geq y$. The aggregate
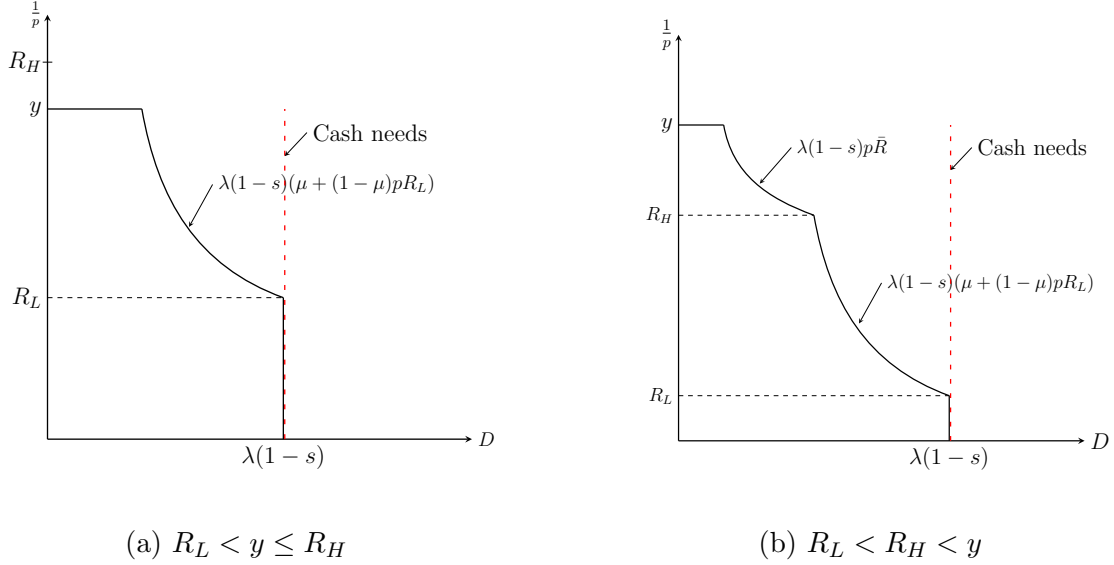
(a) $R_L < y \leq R_H$ (b) $R_L < R_H < y$

Figure 2: Demand of cash

supply function of cash, illustrated in Figure 3, is given by:

$$
S = \begin{cases}
0 & \text{if} & & \frac{1}{p} & < & 1, \\
[0, (1-\lambda)s] & \text{if} & & \frac{1}{p} & = & 1, \\
(1-\lambda)s & \text{if} & 1 & < \frac{1}{p} & < & y, \\
[(1-\lambda)s, s] & \text{if} & & \frac{1}{p} & = & y, \\
s & \text{if} & & \frac{1}{p} & > & y.
\end{cases}
\tag{4}
$$

4. Equilibrium

In this section, we derive the equilibrium allocation. At date $t = 0$, banks decide how much cash and securities to hold, taking into account the expected market price of securities in terms of cash at $t = 1$. Working backward, we first derive the market price of securities at $t = 1$, for a given choice of s, and then we consider the optimal choice of s at $t = 0$.

4.1. Equilibrium price of security at $t = 1$

This sub-section characterizes the market equilibrium price of securities, given a cash level s, chosen at $t = 0$. The price of θ-type securities is denoted by $p^* R_\theta$. First, we prove the following lemma:

Lemma 1. *Given the cash holding level s chosen at $t = 0$, the equilibrium market return on cash $\frac{1}{p^*}$ always satisfies (i) $1 \leq \frac{1}{p^*} \leq y$; and (ii) $R_L \leq \frac{1}{p^*} \leq R_H$.*
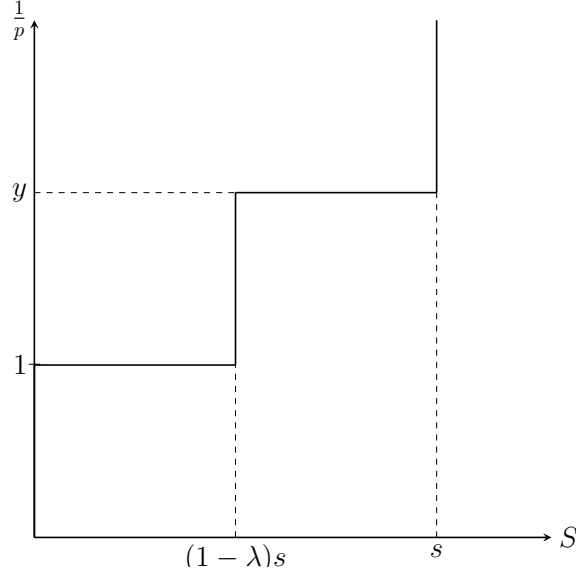
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Figure 3: Supply of cash

Lemma 1 (i) is straightforward from the supply and the demand curve. If the market return on cash $\frac{1}{p}$ is lower than 1, the supply of cash in the market would be 0. On the other hand, if market return on cash is higher than y, there would be no demand for cash in the market. The intuition of lemma 1 (ii) is as following (The formal proof is provided in the appendix.). Note that if $\frac{1}{p} < R_L$, then a bank would be able to continue its non-marketable project even when it holds securities with a low return. Since securities have a higher return than cash, banks would holds only securities, leading to the supply of cash in aggregate being 0 in the market. If $\frac{1}{p} > R_H$, then market return on cash is higher than that of securities even than securities with higher return R_H. Banks would hold only cash, leading to the demand of cash backed by securities being 0. In equilibrium, the market return on cash $\frac{1}{p}$ satisfies $R_L \leq \frac{1}{p^*} \leq R_H$.

Provided lemma 1 holds, the supply of cash in the market is given by only banks that are not affected by the liquidity shock,

$$S = (1 - \lambda)s. \tag{5}$$

When market return on cash, $\frac{1}{p}$ satisfies lemma 1, the demand for cash can be written as:

$$D(p, s) = \begin{cases} \lambda(1 - s) & \text{if} & \frac{1}{p} = R_L, \\ \lambda(1 - s)[\mu + (1 - \mu)pR_L] & \text{if} \quad R_L < \frac{1}{p} \leq R_H. \end{cases} \tag{6}$$

Given the expression for supply (5) and demand (6), we can prove proposition 1, which characterizes the equilibrium prices of securities, taking as given the cash level s.

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Proposition 1. *Under $pR_L \leq 1 \leq pR_H$, given the level of cash holding s, the equilibrium $p^* (s)$ is determined as follows:*

1. *If $S \geq D (p = 1)$, then $p^* (s) = 1$. The price of security is its fundamental value.*

2. *If $S < D (p = 1)$, then $p^* (s)$ is determined by $S = D$ yielding*

$$p^* (s) = \frac{1}{R_L} \left[\frac{(1 - \lambda) s}{\lambda (1 - s)} - \mu \right] \frac{1}{1 - \mu}.$$

   The price of security is below its fundamental value.

3. *If $s = \lambda$ and $R_L \geq 1$, p^* can be any value in $\left[\frac{1}{R_L}, 1 \right]$, i.e. any value between $p^* (s)$ determined by $S = D$ and 1.*

$D (p = 1)$ is the demand of cash backed by the fundamental value of security. If supply of cash is higher than $D (p = 1)$, the price of security is its fundamental value. Inversely, if supply of cash is lower than $D (p = 1)$, the price of security is determined by the amount of cash in the market and is lower than its fundamental value ("cash-in-the-market" pricing).

4.2. Individual Portfolio Choice at $t = 0$

Each bank decides how much cash to hold at $t = 0$ to maximize its expected profit. Given θ, the marginal return on cash depends on whether it can obtain necessary amount of cash or not when it is hit by liquidity shock. In other words, the marginal return on cash depends on whether or not $s + (1 - s) pR_\theta \geq 1$, equivalently, $pR_\theta \geq 1$. Since $pR_L \leq 1 \leq pR_H$ in equilibrium, it depends on which type of security the bank holds. When the bank holds H-type, it is not liquidity constrained, so the marginal return on cash is simply

$$\frac{1}{p} - R_H;$$

the gross return on one additional unit of cash minus the opportunity cost of that additional unit.

If the bank holds L-type, then we have two cases upon the price of L-type security pR_L. When $pR_L = 1$, the bank is not liquidity constrained, so the marginal return on cash is similar to above and can be written as

$$\frac{1}{p} - R_L.$$

When $pR_L < 1$, the bank is liquidity constrained. In this case, the marginal return is given by

$$\left[\lambda y + (1 - \lambda) \frac{1}{p} \right] - \left[\lambda pR_L y + (1 - \lambda) R_L \right].$$

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The first term in brackets corresponds to the bank's marginal return on cash: if the bank is affected by a liquidity shock, which happens with probability λ, then the marginal return on cash is equal to the marginal return on the project, y, while if the bank is not affected by the liquidity shock, then the cash can be used to purchase securities, with a marginal return of $\frac{1}{p}$. An additional unit of cash also carries an opportunity cost, corresponding to the return of the securities that could have been purchased instead. This is captured by the second term in brackets: if the bank is affected by a liquidity shock, it would sell the additional securities at price pR_L to acquire cash, while if the bank is not affected by the liquidity shock the additional securities would return R_L.

Using these expressions, we can derive the expected marginal (net) return on cash conditional on the price of securities p, which is given by

$$\frac{\partial \pi}{\partial s} = \begin{cases} \frac{1}{p} - \bar{R} & \text{if} \quad pR_L \;\; = \;\; 1 \\ \left(\frac{1}{p} - \bar{R}\right) + (1 - \mu)\lambda\left(y - \frac{1}{p}\right)(1 - pR_L) & \text{if} \quad pR_L \;\; < \;\; 1 \end{cases} \tag{7}$$

When $pR_L < 1$, the expected marginal return on cash is the sum of two components. The first term, $\left(\frac{1}{p} - \bar{R}\right)$, represents the pecuniary or "fundamental" net return on cash, which can be negative. The second term represents the additional value of cash that comes from its use in continuing the long-term project when a liquidity shock hits. The relative size of these two terms will determine whether there is cash hoarding or cash-in-the-market pricing in equilibrium.

The equilibrium cash holding level s^* given p must maximize the expected return on cash. From this together with the proposition 1, we can characterize market equilibria, represented by the pair of the price p^* and the level s^* of individual cash holding.
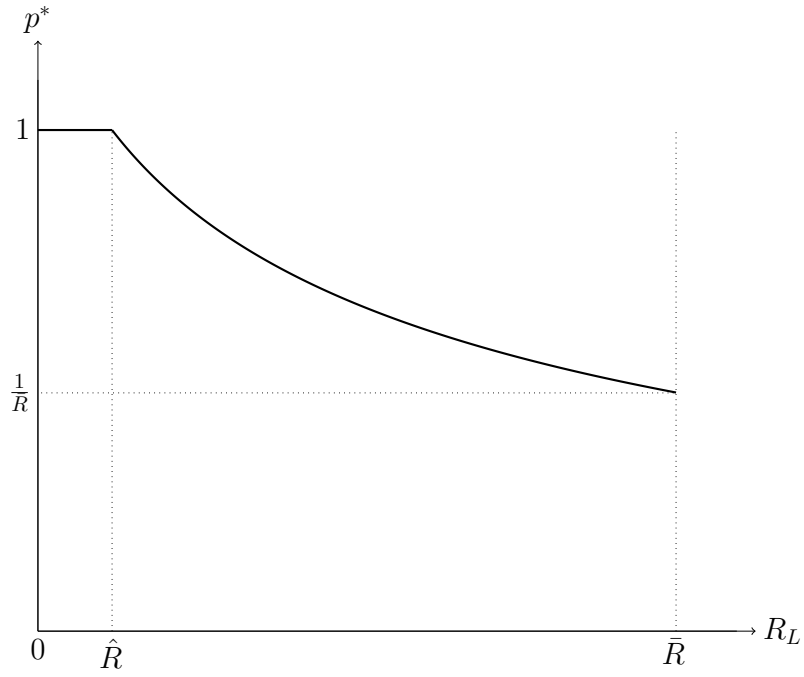
Proposition 2. *For each $R_L \le R_H$, there exist a unique equilibrium:*

1. *When $R_L = R_H = \bar{R}$, there exist an equilibrium with $s^* = \lambda$ and $p^* = \frac{1}{\bar{R}}$. (optimal)*

2. *When $\hat{R} < R_L < \bar{R}$, there exist an equilibrium with $s^* < \lambda$ and $p^* < 1$. (cash-in-the-market pricing) s^* and p^* are determined by*

$$s^* \;\; = \;\; \frac{\lambda\left(\mu + (1 - \mu)pR_L\right)}{\lambda\left(\mu + (1 - \mu)pR_L\right) + (1 - \lambda)} \tag{8}$$

$$\frac{1}{p^*} \;\; = \;\; \bar{R} - (1 - \mu)\lambda\left(y - \frac{1}{p^*}\right)(1 - p^*R_L) \tag{9}$$

3. *When $R_L < \hat{R}$, there exist an equilibrium with $s^* = 1$ and $p^* = 1$. (cash hoarding, with the price being the fundamental value)*

(a) Equilibrium p^*
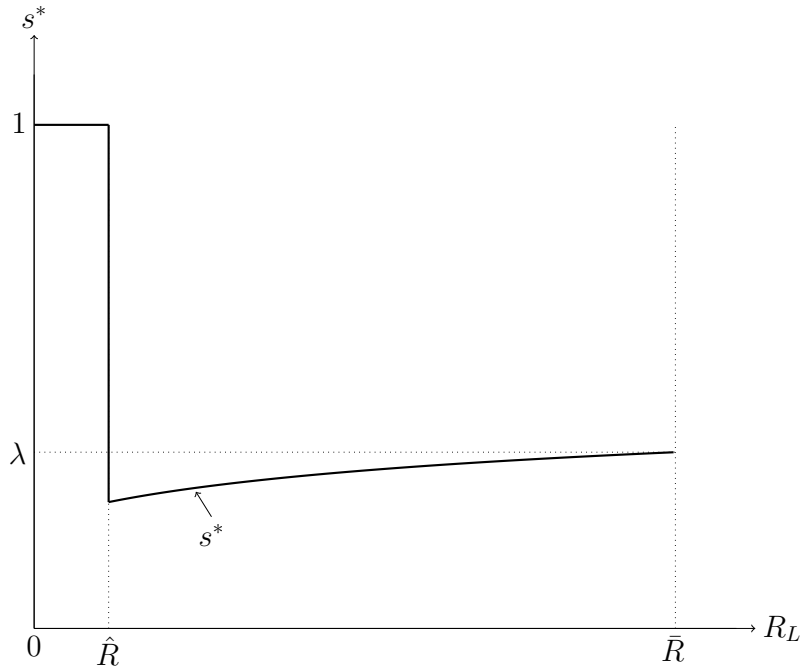
(b) Equilibrium cash holding level s^*

Figure 4: Equilibrium price and cash holding level

The threshold level $\hat{R}$ of R_L is given by

$$\hat{R} \equiv 1 - \frac{\bar{R} - 1}{(1 - \mu)\,\lambda\,(y - 1)}. \tag{10}$$

The proof is provided in the appendix.

Figure 4 illustrates the equilibrium price and cash level upon the value of security with low quality R_L. It is noteworthy that λ is the optimal level of cash in the economy. Banks need cash only for the purpose of continuing project hit by liquidity shock in our model. Proportion λ of banks is affected by liquidity shock, in which case 1 unit of cash should be injected in order to continue the project in full scale. Thus total cash needed in the economy is λ. This is the case independent of the value of the security with low quality, R_L, whereas market equilibrium level of cash depends upon R_L.

There exist a trade off when the bank holds securities in its portfolio. Compared to cash, marketable securities exhibit an advantage in the sense that they generate an expected return higher than cash ($\bar{R} > 1$) and can be used to obtain cash if needed by selling them in the market (or equivalently by pledging them as collateral and borrowing cash). However, increasing the amount of securities in one's portfolio increases as well the risk the project hit by liquidity shock cannot be continued in full scale when the security is of low quality. In that case, its foregone return y is higher than expected return on securities $\bar{R}$. This risk becomes higher as R_L is low. As a consequence, market equilibrium is characterized upon the fundamental value of the security with low quality.

When R_L is low such that $R_L < \hat{R}$, market allocation results in an equilibrium with cash hoarding.[7] When it exists, this equilibrium corresponds to a situation where the return on cash that comes from its use to continue the non-marketable project exceeds, in equilibrium, the return from holding securities. In such a situation, the interbank market functions very poorly as a way to redistribute cash between banks. As a consequence, the interbank market freezes; all banks hold as much cash as they need and no trade takes place. Since there is cash hoarding in this equilibrium, the price of securities always corresponds to their fundamental value.

When R_L is high such that $\hat{R} < R_L < R_H$, the equilibrium level of cash holding by banks is lower than the optimal amount, $s^* < \lambda$. When R_L is high, banks are less concerned by liquidity shock considering that they can come up with cash needs by selling their securities. Banks thus are holding cash less than socially optimal. In this equilibrium banks do not internalize the effect their cash-holding decision has on the equilibrium price of securities. Because banks are holding so little cash, equilibrium p^* is always lower than 1. In other words, there is cash-in-the-market pricing. This, in turns, reduces the extent to which banks hit by a liquidity shock can continue their project.

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[7]Such an equilibrium may not exist, depending on the value of other parameters and if $\hat{R}$ is lower than 0.

5. Comparative statics

5.1. *Dispersion of return on securities and cash holding*

Our analysis demonstrated that the quality of marketable securities has an impact on the banks' behavior on cash holding. We investigate more on their relationship taking dispersion of return on securities as a measure of the quality of securities. The risk a bank cannot hedge liquidity shock becomes higher as dispersion of return on marketable securities becomes higher because it becomes higher the probability that the value of the security is not enough to obtain the amount of cash needed.

We use the coefficient of variation, c_v, as our measure of dispersion, which is defined as standard deviation of returns on securities divided by their expected return. Precisely,

$$c_v = \frac{\sqrt{\mu R_H^2 + (1 - \mu) R_L^2}}{\bar{R}}$$

Given $\bar{R} \equiv \mu R_H + (1 - \mu) R_L$, c_v can be rewritten as

$$c_v = \sqrt{\frac{1 - \mu}{\mu}} \left(1 - \frac{R_L}{\bar{R}} \right) \tag{11}$$

In order to analyze the sole effect driven by changes in dispersion on the amount of cash held by banks, we take expected return on securities $\bar{R}$ as given and focus only dispersion between return on the securities with high quality (R_H) and that with low quality (R_L). Using proposition 2, we obtain the following proposition for $\bar{R} \in (1, y)$, which is relevant range of $\bar{R}$:

Proposition 3. *Given a constant $\bar{R} \in (1, y)$,*

1. *if dispersion of return on securities c_v is lower than $\hat{c}_v$, then there exhibit a negative relationship between dispersion of return and equilibrium level of cash holding;*

2. *if dispersion of return on securities c_v is higher than $\hat{c}_v$, banks hold exclusively cash without securities independent of dispersion.*

The threshold level of dispersion, $\hat{c}_v$ is given by

$$\hat{c}_v = \frac{\bar{R} - 1}{\bar{R}} \sqrt{\frac{1 - \mu}{\mu}} \left[1 + \frac{1}{(1 - \mu) \lambda (y - 1)} \right] \tag{12}$$

The proof is relegated to the appendix. Figure 5 (a) illustrates the relationship between dispersion and the cash holding level described in proposition 3. Interestingly, we have non trivial non monotonous impact of the level of dispersion. When dispersion is low, as dispersion grows banks hold less cash. However, when dispersion becomes higher exceeding a certain threshold, $\hat{c}_v$, banks hold only cash to hedge their liquidity risk.

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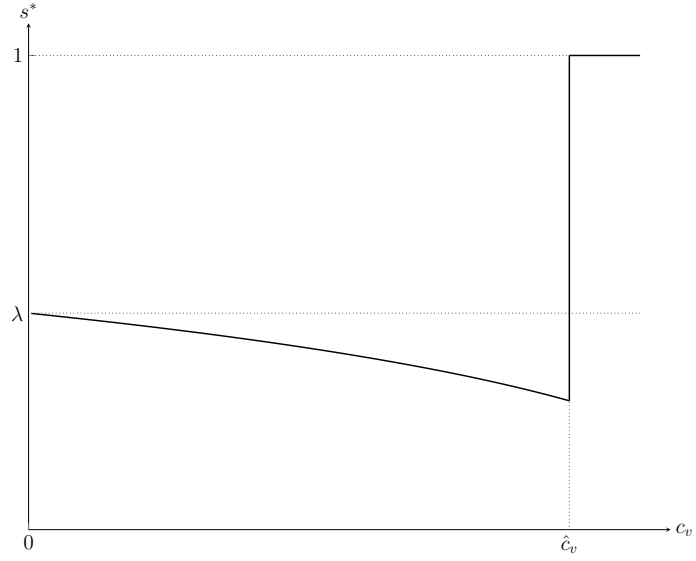
(a) Case with benchmark model $(x = 1)$
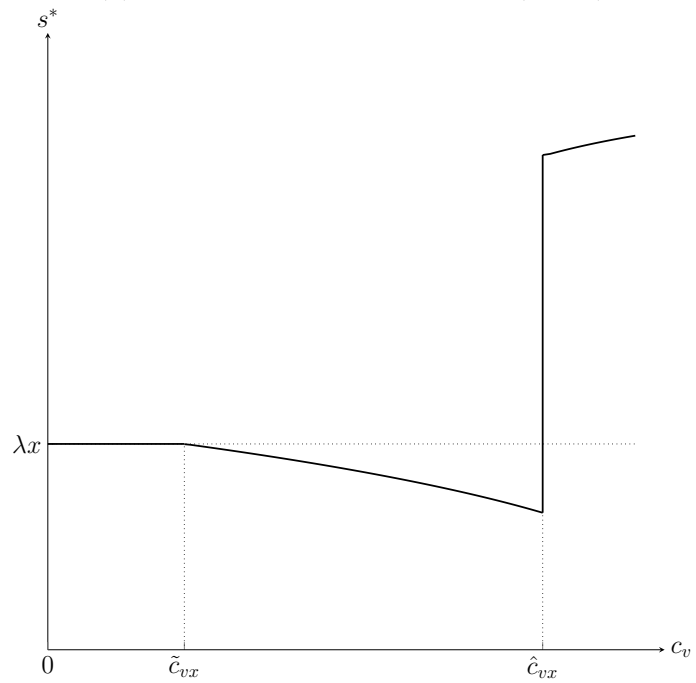
(b) Case with $x \leq 1$

Figure 5: Effect of changes in dispersion of return on securites

Until now, we have assumed that bank needs the amount of liquidity 1 to continue its project when it is affected by liquidity shock. We can extend our analysis to the case more general, with the amount of cash needed in the case of liquidity shock, *i.e.*, $x \leq 1$. Analysis analogous with the case with $x = 1$ gives the following proposition[8]

Proposition 4. *Given a constant* $\bar{R} \in \left(1, \frac{y}{x}\right)$,

1. *if dispersion of return on securities* c_{vx} *is low, i.e., lower than* $\tilde{c}_{vx}$, *then the equilibrium level of cash held by banks is not affected by dispersion and always remains unchanged at* λx *(optimal);*

2. *if dispersion of return on securities* c_{vx} *intermediate, i.e., lies in between* $\tilde{c}_{vx}$ *and* $\hat{c}_{vx}$, *then there exhibit a negative relationship between dispersion of return and equilibrium level of cash holding;*

3. *if dispersion of return on securities* c_v *is high, i.e., higher than* $\hat{c}_{vx}$, *there exhibit positive relationship between dispersion of return and level of cash holding.*

The threshold level of dispersion, $\tilde{c}_{vx}$ *and* $\hat{c}_{vx}$ *is given by*

$$\tilde{c}_{vx} = \sqrt{\frac{1-\mu}{\mu}} \left[1 - \frac{1}{\bar{R}} \left(\frac{x-\lambda x}{1-\lambda x}\right) \left\{ \frac{\bar{R} - (1-\mu)\lambda \left(1 - \frac{x-\lambda x}{1-\lambda x}\right)\frac{y}{x}}{1 - (1-\mu)\lambda \left(1 - \frac{x-\lambda x}{1-\lambda x}\right)} \right\} \right] \tag{13}$$

$$\hat{c}_{vx} = \frac{\bar{R}-1}{\bar{R}} \sqrt{\frac{1-\mu}{\mu}} \left[1 + \frac{1}{(1-\mu)\lambda \left(\frac{y}{x}-1\right)} \right] \tag{14}$$

The proof is analogous to that of proposition 3 and is based on the appendix A.

Figure 5 (b) describe the proposition 4. We can observe more clear non monotonous relationship between banks' cash holding level and dispersion of return on securities. We can easily verify that $\tilde{c}_{vx}$ in (13) and $\hat{c}_{vx}$ in (14) become 0 and $\hat{c}_v$ in (12) respectively when $x = 1$. Proposition 3 is thus a particular case of proposition 4. Our proposition about relationship between dispersion of returns on securities and cash holding behavior by banks are therefore robust.

The threshold $\hat{c}_v$ (or $\hat{c}_{vx}$ in general case with $x \leq 1$) represents the value of dispersion above which banks changes their behavior. In other words, above this threshold of dispersion, they shift their behavior from holding too little cash to holding too much cash, compared to socially optimal level, on the one hand, and the relationship between dispersion of returns and banks' cash holding is changed from negative to positive. This threshold value depends upon several variables according to equation (12) and (14). Proposition 5 follows:

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[8]Full analysis on the case with $x \leq 1$ is presented in the appendix A.

Proposition 5. $\hat{c}_v$ *(or $\hat{c}_{vx}$ in general case with $x \leq 1$) increases*

1. *as $\bar{R}$ increases; and*

2. *as λ decreases.*

In a word, the threshold of dispersion level above which the relationship between dispersion and cash holding becomes higher as expected return on securities increases, as the probability of liquidity shock decreases.

5.2. *Expected return on securities and equilibrium level of cash*

Now, we analyze the impact that changes in expected return on securities $(\bar{R})$ have on cash holding behavior of bank. Similar to the previous analysis, In order to control the impact driven by changes in dispersion of return on securities and to focus on pure impact of change in $\bar{R}$, we conduct a comparative statics taking dispersion (c_v) as given. Proposition 6 follows.

Proposition 6. *For a relevant range of $\bar{R} \in (1, y)$,*

1. *if there is no dispersion of return on securities, changes in expected return has no impact on the equilibrium level of cash holding. it is always $s^* = \lambda$ independent of expected return on securities.*

2. *If dispersion of return on securities is positive, there is a (weakly) negative relationship between expected return on securities and the equilibrium level of cash holding. Precisely*

   (a) *if expected return $\bar{R}$ is high, i.e., $\bar{R} > \hat{\bar{R}}$ then, bank increases equilibrium level of cash holding as expected return on securities decreases.*

   (b) *if expected return $\bar{R}$ is low, i.e., $\bar{R} \leq \hat{\bar{R}}$ banks increases their cash holding level substantially to $s^* = 1$ at $\bar{R} = \hat{\bar{R}}$ then equilibrium level of cash holding remains unchanged irrespective of expected return.*

   (c) *The threshold level of expected return at which cash holding level jumps substantially becomes lower as dispersion of returns on securities becomes lower.*

The threshold level of expected return, $\hat{\bar{R}}$ is given by

$$\hat{\bar{R}} = \frac{1 + (1 - \mu)\,\lambda\,(y - 1)}{1 + (1 - \mu)\,\lambda\,(y - 1)\left(1 - c_v\sqrt{\frac{\mu}{1-\mu}}\right)} \tag{15}$$
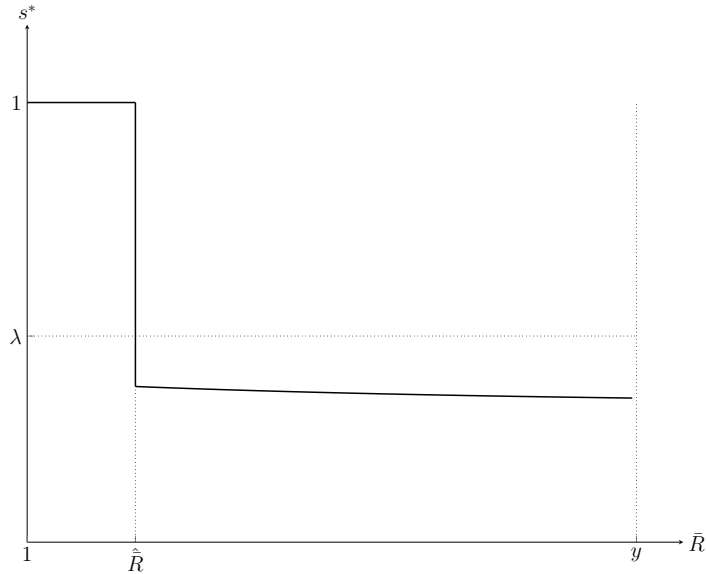
Proof is relegated to the appendix.

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Figure 6: The effect of changes in expected return on securites

Figure 6 illustrates proposition 6. It is noteworthy that unlikely to the relationship between cash holding level of banks and dispersion of return on securities, there exist a monotonous relationship between cash holding level and expected return on securities. The behind idea is straightforward. Negative relationship between expected return and cash holding level is driven by the relative return of securities compared to cash on the one hand and by the precautionary motive on the other hand. As expected return on securities decreases, banks decreases the amount of securities in their portfolio, which in turn increases the amount of cash. Moreover, Decrease in expected return on securities reduces the (expected) amount of cash that can be obtained by using the security as a collateral when they affected by liquidity shock. This thus increases the risk of liquidation of investment project. Facing increase in risk of liquidation, banks accumulate more cash to hedge.

5.3. *Probability of liquidity shock and equilibrium level of cash*

Now, we analyze how the change in probability of liquidity shock affects the individual bank's decision on liquidity holding. Similar to the previous analyses, we investigate on the effect of change in probability λ of liquidity shock on the equilibrium level of cash holding (s^*) taking expected return on securities and their dispersion as given. Proposition 7 follows.

Proposition 7. *1. Increase in probability of liquidity shock results in an increase in equilibrium cash holding level of banks.*

   2. Concrete path of cash level held by banks upon the probability of liquidity shock depends on the dispersion of return on securities:
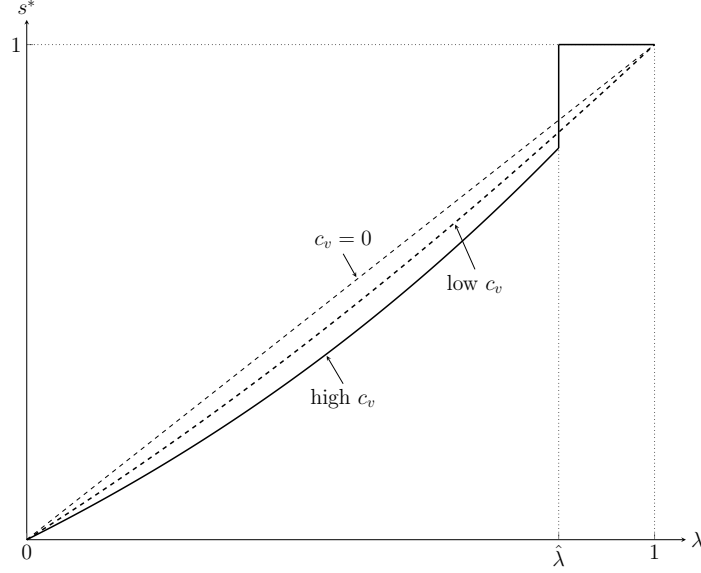
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Figure 7: The effect of increase in probability of liquidity shock

(a) When the dispersion of return is 0, the equilibrium level of cash holding is always λ;

(b) When the dispersion is low such that $0 < c_v \leq \sqrt{\frac{1-\mu}{\mu}}\left(1 - \frac{1}{R}\right)$, the equilibrium level of cash holding increases as λ increases but always lower than λ;

(c) When the dispersion is high such that $c_v > \sqrt{\frac{1-\mu}{\mu}}\left(1 - \frac{1}{R}\right)$,

    i. if λ is lower than $\hat{\lambda}$, the equilibrium level of cash holding increases as λ increases but always lower than λ;

    ii. if λ is higher than $\hat{\lambda}$, the equilibrium levle of cash holding is 1.

The threshold level of $\hat{\lambda}$ is defined as

$$\hat{\lambda} \equiv \frac{\bar{R} - 1}{(1 - \mu)(y - 1)\left(1 - \bar{R}\left(1 - c_v \sqrt{\frac{\mu}{1-\mu}}\right)\right)} \tag{16}$$

Proof is analogous to that of proposition 3 and 6.

Figure 7 illustrates the above proposition. Banks rise their level of cash holding as the probability of liquidity shock increases.

6. Policy interventions for cash-in-the-market pricing situations

We analyze whether an intervention can improve welfare compared to market outcome. In this section, we focus on the situation with cash-in-the-market pricing.

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6.1. *The planner's problem: Optimal allocation between cash and securities*

For this purpose, we need benchmarks to compare and we determine the date-0 optimal allocation from the point of view of a social planner. Since all agents are risk neutral, we define welfare as total expected output. The social planner aims to maximize social welfare by choosing the cash level s that individual banks hold at $t = 0$ and by redistributing cash among banks at $t = 1$, so that the banks subject to a liquidity shock have the cash they need to continue their project.

The total output of economy is given by

$$W(s) = \max\left\{1, 1 - \lambda + \lambda s + C\right\} y + \left[(1 - \lambda) s - C\right] + (1 - s)\bar{R}, \tag{17}$$

subject to the feasibility constraint

$$C \leq (1 - \lambda)s, \tag{18}$$

where C is defined as the aggregate amount of cash transferred to the bank with a liquidity shock and $(1 - \lambda)s$ stands for the total amount of cash available for transfer. The first term in (17) is the return from projects that mature. A proportion $1 - \lambda$ of banks are not hit by the liquidity shock and their project will mature yielding y. In addition, a proportion λ of banks are affected by the liquidity shock. Each of these banks holds an amount s of cash and obtains additional cash from the social planner. In aggregate, a proportion $1 - \lambda + \lambda s + C \leq 1$ of project, each yielding y, will be continued. The second term represents leftover cash, which cannot be negative, and the third term captures the return of tradable securities in the economy.

The amount of cash $1 - s$ is needed to a bank with liquidity shock in order to continue its project in full scale. The total amount of cash needed for the banks hit by liquidity shock is therefore $(1 - s)\lambda$. Since the expected return of the project, y, is greater than the expected return on securities, $\bar{R}$, which is itself greater than the return on cash, 1, the one extra unit of cash scarifying 1 unit of securities increases social welfare only when it can be injected into the project hit by liquidity shock. Therefore, socially optimal level of cash is the exact amount such that the total amount of cash available for transfer is same as that needed for banks hit by liquidity shock, In other words, s such that

$$(1 - \lambda)s = (1 - s)\lambda$$

which yields $s^{FB} = \lambda$. To put it differently, this is the amount of cash that needed to continue all the projects hit by liquidity shock.

Next we consider the problem of the social planner who maximizes welfare in the economy under the constraint that a bank cannot receive more cash than the value of the securities it holds. For example, banks may be able to hide the cash they hold and the

planner can only transfer cash if banks are compensated for the cash they give up. Studying the allocation of the constrained planner is useful to distinguish the inefficiency related to the redistribution of cash in the interbank market from the inefficiency related to the choice of cash held by banks at date 0. This benchmark is also useful when we consider the optimal allocation with the social planner who is not allowed to make loss.

The constrained-optimal, or second-best, level of cash is denoted s^{SB}. A share μ of cash-poor banks hold H-security, which have a value greater than 1 by assumption, so they can receive the amount $1 - s$ of cash against (some of) their securities. However, a share $1 - \mu$ of bank with liquidity shock hold L-security, which can have value lower than 1. Let c_θ denote the transfer to banks holding θ-security, $\theta \in \{L, H\}$. The constraint that the social planner faces for banks holding L-security is therefore

$$c_L \leq (1 - s)R_L.$$

Note that the planner would never transfer more than $1 - s$, so the above constraint binds only if $R_L < 1$. As a result, the constrained optimal level of cash remains unchanged when $R_L \geq 1$. However, when $R_L < 1$, the aggregate transfer constraint is added for consideration, which is

$$C \leq \lambda (1 - s) (\mu + (1 - \mu) R_L) \tag{19}$$

With the feasibility condition, the amount of aggregate transfer of cash should satisfy

$$C \leq \min \{(1 - \lambda) s, \, \lambda (1 - s) (\mu + (1 - \mu) R_L)\}. \tag{20}$$

To find the marginal contribution of cash to output, $\frac{\partial W}{\partial s}$ when $R_L < 1$, we can plug (20) into (17) and differentiate with respect to s, which yields

$$\frac{\partial W}{\partial s} = \begin{cases} y - \bar{R} & \text{if} \quad s \ < \ \hat{s}, \\ \lambda (1 - \mu) (1 - R_L) y - \lambda \mu (R_H - 1) - (1 - \lambda) (\bar{R} - 1) & \text{if} \quad s \ > \ \hat{s}. \end{cases} \tag{21}$$

where

$$\hat{s} \equiv \frac{\lambda (\mu + (1 - \mu) R_L)}{\lambda (\mu + (1 - \mu) R_L) + 1 - \lambda} < \lambda \equiv s^{FB}. \tag{22}$$

$\hat{s}$ represents the maximum level of cash such that there is no cash left over after transfers if $R_L < 1$. It is noteworthy that $\hat{s}$ is lower than first-best level of cash due to the transfer constraint (19). For $R_L < 1$, additional cash always increases output when $s < \hat{s}$. Indeed, when $s < \hat{s}$ an additional unit of cash can be used to continue a larger share of the project without resulting in any leftover cash after transfers. However, when $s > \hat{s}$ given $R_L < 1$, the sign of the marginal output of cash depends on the value of R_L as is given by the second line of (21). Its first term stands for additional output thanks to the continuation of more proportion of project for a bank hit by liquidity shock and with security L. This term is positive when $R_L < 1$ and increases as R_L falls. The second and third term represent forgone

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output related to holding more cash without being injected to the project, respectively for the banks affected by liquidity shock and with security H, and for those without liquidity shock. We can derive the value of R_L such that the first term equalize the the sum of second and third term, which is same as $\hat{R}$. If $R_L > \hat{R}$, then $\partial W/\partial s < 0$, so the planner wants to minimize the amount of cash held, conditional on continuing as much of the non-marketable projects as possible. In contrast, if $R_L < \hat{R}$, then $\partial W/\partial s > 0$, so the planner wants to maximize the amount of cash.

These results are summarized in the following proposition 8:

Proposition 8. *1. When $R_L \geq 1$, the additional constraint faced by the planner does not bind so that $s^{SB} = s^{FB} = \lambda$.*

2. When $\hat{R} < R_L < 1$, the constrained optimal level of cash s^{SB} is $\hat{s}$ ($< s^{FB}$).

3. When $R_L = \hat{R}$, the constrained optimal level of cash s^{SB} can be any value between $[\hat{s}, 1]$.

4. When $R_L < \hat{R}$, the constrained optimal level of cash s^{SB} is 1 ($> s^{FB}$).

The constrained optimal level of cash depends on R_L, the low return for the securities. When R_L is higher than 1, the planner can achieve the first-best because the constraint does not bind. When R_L is smaller than 1, the planner can no longer transfer the optimal amount of cash. The planner chooses to reduce s for two reasons: First, a reduction of s is necessary to make sure that there is no unused cash after transfers. Second, reducing s allows banks to hold more securities, which can be used to acquire more cash when the return on the securities is low. For these reasons, s^{SB} decreases as R_L decreases until R_L becomes smaller than $\hat{R}$ at which point it jumps to 1. The intuition for this result is that when R_L is so low, the loss associated with liquidating project for the banks with L-type securities outweighs the gain from reducing cash remaining untransferred. Hence, the planner prefers that banks hold sufficient cash to continue the project without needing a transfer that is constrained by the value of the securities.

6.2. *Comparison between market outcome and optimal allocation*

Figure 8 illustrates Optimal (s^{FB}) and constrained optimal(s^{SB}) level of cash holding compared to market equilibrium level of cash (s^*). An equilibrium with cash hoarding can only exist when R_L is lower than $\hat{R}$.[9] When it exists, this equilibrium corresponds to a situation where the return on cash that comes from its use to continue the non-marketable project exceeds, in equilibrium, the return from holding securities. In such a situation, the interbank market functions very poorly as a way to redistribute cash between banks. As a

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[9]Such an equilibrium may not exist, depending on the value of other parameters and if $\hat{R}$ is lower than 0.
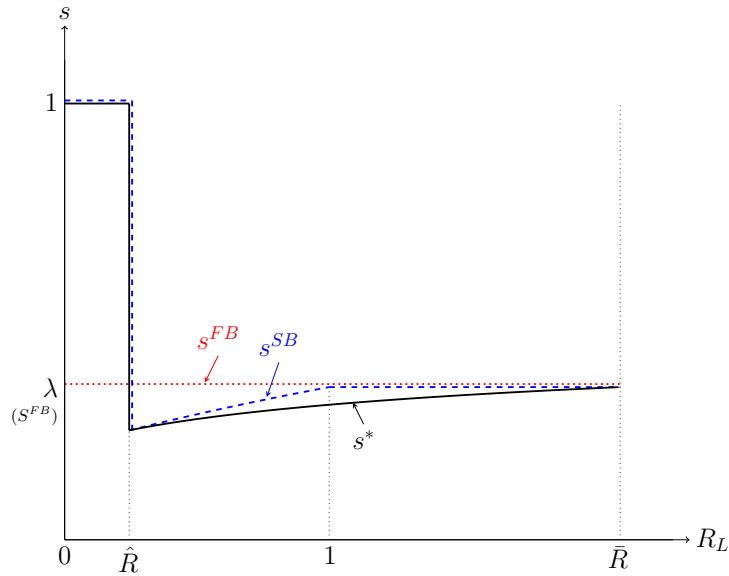
Figure 8: Optimal level of cash holding

consequence, the interbank market freezes; all banks hold as much cash as they need and no trade takes place.

Since there is cash hoarding in this equilibrium, the price of securities always corresponds to their fundamental value. The level of cash is constrained optimal.

When $\hat{R} < R_L < \bar{R}$, the equilibrium level of cash holding by banks is lower than the optimal amount, $s^* < \lambda$, and even lower than the constrained optimal amount, $s^* < s^{SB}$. In this case, p^* and s^* are determined by (8) and (9). Because banks are holding so little cash, equilibrium p^* is always lower than 1. In other words, there is cash-in-the market pricing. As already discussed, in this equilibrium banks do not internalize the effect their cash-holding decision has on the equilibrium price of securities. Banks hold too little cash, which implies that the value of securities is low because of cash-in-the-market pricing. This, in turns, reduces the extent to which banks hit by a liquidity shock can continue their project.

The threshold $\hat{R}$ represents the value of R_L below which individual banks shift their behavior from holding too little cash to holding too much cash, compared to the socially optimal level. Equation (10), shows that this threshold increases as the proportion of low quality securities, $1 - \mu$, and the proportion of projects hit by liquidity shock, λ, increases. If both of these values are high, then the value of R_L below which banks hold too much cash becomes relatively high. Indeed, if μ tends to zero and λ tends to 1, then $\hat{R}$ tends to 1.

In words, if the share of securities with a low return and the share of non-marketable projects that are hit by a liquidity shock are high the interbank market may not be very effective at redistributing cash among banks. The market is likely to freeze as banks attempt

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to self-insure.

6.3. *Liquidity regulation*

When the economy faces cash-in-the-market pricing, a liquidity requirement, such as the LCR, can improve welfare. For the sake of illustration, consider the case with $\hat{R} < R_L < R_H$, where the market equilibrium exhibits cash-in-the-market pricing ($s^* < \lambda$). In equilibrium, welfare is given by:

$$W\left(s\right) \;\; = \;\; \left(1 - \lambda + \lambda s + C^m\right) y + \left[\left(1 - \lambda\right) s - C^m\right] + \left(1 - s\right) \bar{R},$$

where C^m is the aggregate amount of cash that banks faced with a liquidity shock can obtain in the market. It is easy to verify that $C^m = \left(1 - \lambda\right) s$. Plugging this expression into $W\left(s\right)$ yields total welfare at the market equilibrium cash level s^*

$$W\left(s^*\right) \;\; = \;\; \left(1 - \lambda + s^*\right) y + \left(1 - s^*\right) \bar{R}$$

The marginal contribution of cash to welfare, $\frac{\partial W}{\partial s}$, is $y - \bar{R}$, which is always positive. Therefore, requiring banks to hold more cash than s^* improves welfare.

Not surprisingly, the solution for the welfare maximizing level of cash that banks should be required to hold is same as the level of cash chosen by the constrained social planner. This is summarized in the next proposition.

Proposition 9. *1. When the economy displays cash-in-the-market pricing, liquidity requirements, such as the LCR can improve welfare.*

2. In such a situation, liquidity requirement can restore first-best when $1 \leq R_L < R_H$ and can restore contained optimal level of welfare (second-best) when $\hat{R} < R_L < 1$.

By forcing banks to hold more liquidity, a requirement like the LCR increases the value of securities and permits more long-term projects to be continued, resulting in higher welfare.

An interesting implication of this result is that the appropriate level of the LCR should depend in part on the dispersion of the return of marketable securities. More precisely, the constrained optimal level of cash becomes lower as the dispersion of the return on securities is greater. Since the dispersion of returns is likely to be greater during periods of crisis, our model suggests that the liquidity requirement should be lower in times of stress than in good time.

6.4. *Announcement of asset purchase by the central bank*

Another policy intervention that can improve welfare when the economy faces cash-in-the-market pricing is for the central bank to announce at $t = 0$ that it will purchase the securities at their fundamental value at $t = 1$, which implies that $p = 1$.

To analyze the effect of asset purchase by the central bank, we slightly modify our basic model. We suppose that at $t = 0$ the central bank levies a tax $\tau < 1$ on banks in the form of cash. Hence, the amount of cash held by banks is $s \in [0, 1 - \tau]$ and the amount of securities is $1 - s - \tau$. At $t = 1$, banks can obtain the cash by trading their securities either with other cash-rich banks at market price or with the central bank at the price $p = 1$. The central bank can purchase securities until it runs out of cash. The central bank chooses τ to maximize social welfare. The following proposition describes an equilibrium:

Proposition 10. *1. When the economy displays cash-in-the-market pricing, the central bank can improve welfare by levying a tax $\tau > 0$ at $t = 0$ and using the proceeds to purchase securities at their fundamental value ($p = 1$) at $t = 1$.*

(a) When $1 \leq R_L < \bar{R}$, such intervention with $\tau = \lambda$ can restore the first-best allocation.

(b) When $\hat{R} < R_L < 1$, such intervention with $\tau = s^{SB}$ can restore the constrained optimal allocation.

2. In both cases, banks choose to hold no cash $s^* = 0$ in equilibrium. The market for securities is inactive and all trading occurs with the central bank.

Proof. Here, we will demonstrate that when $\hat{R} < R_L < 1$, intervention with $\tau = s^{SB}$ can restore the constrained optimal allocation. The other case can be proved in a similar way. When $\hat{R} < R_L < 1$, we know that the constrained optimal allocation is attained when the total cash in the economy is s^{SB} and if all of the cash redistributed to banks that need it. Thus, it is sufficient to demonstrate that banks have no incentive to deviate from $s^* = 0$ and that the cash held by the central bank will be transferred to the banks with liquidity shock. First, note that this policy is feasible, since $\tau = s^{SB}$ and the central bank sets $p^{CB} = 1$.

If the market equilibrium price is $p^* < 1$, then all banks that need cash sell their securities to the central bank. Since s^{SB} is the maximum amount of cash that is demanded when the securities are traded at their fundamental value, all demand for cash can be satisfied by the central bank and there is no trade in the market. Therefore, $p^* < 1$ cannot be an equilibrium.

If the market equilibrium price is $p^* = 1$, then banks with liquidity shock are indifferent between selling their securities to the central bank and selling them in the market. However, when $\hat{R} < R_L$ and $p^* = 1$, the marginal return on cash expected at $t = 0$ is always negative. As a consequence, banks will not hold any cash. Therefore, $s^* = 0$ is an equilibrium given $\tau = s^{SB}$ and $p^{CB} = 1$. In such an equilibrium, all the cash s^{SB} is transferred to banks with liquidity shock since it is equal to demand for cash by these banks. $\square$

The central bank's intervention described in the proposition above resembles the Quantitative Easing (QE) polices that several central bank conducted during the recent crisis

along some dimensions. These policies can increase the value of securities by "flooding the market" with reserves. This increase brings the value of the securities back to their fundamental value. Flooding the market with reserves typically depress activity in the interbank market and this happens in the model too. With the cash injections from the central bank, there is no need for banks to trade with each other.

7. Policy intervention for cash hoarding situations.

When $R_L < \hat{R}$, there is cash hoarding. The decentralized individual choice of cash holding is clearly higher than the optimal amount as defined by the first-best. However, our result shows that the market equilibrium is constrained optimal. Indeed, in such situation the value of cash to continue the long-term project is so high that it more than compensate for the fact that cash has a lower expected "fundamental" return than securities. As a result, the central bank cannot improve welfare except, unless it is willing to purchase securities at a value that is greater than their fundamental value, which would lead to losses.

7.1. *Mutual insurance*

In this type of situation, an insurance scheme among banks could restore the first best. If banks can commit to share the available cash, then they face the same problem as an unconstrained planner. This type of insurance mechanisms can be costly to implement and difficult to enforce, so that they might not be set up ex ante. This might be the case for example, if the economy is rarely expected to be in a situation where banks would want to hold so much cash.

That said, the type of clearing house arrangements that arose during banking panics, described in Gorton (1985), have characteristics that are similar to an insurance mechanism that would restore the first best in our model. In response to a panic, clearinghouse member banks would join together to form a new entity overseen by the Clearinghouse Committee. As Swanson (1908) put it, describing the crisis of 1860, there was "the virtual fusion of the fifty banks of New York into one central bank" (p. 221).[10]

7.2. *Central bank's lending against non-marketable project*

It is also possible for a central bank to achieve the unconstrained optimal allocation if it can lend against the return of the non-marketable long-term project. While this result requires assuming that the central bank can choose actions that are not permissible to market participants, it is consistent with empirical evidence.

In practice, some central banks do lend against such assets. In Europe, assets which do not have an International Securities Identification Number (ISIN) are typically very

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[10]See Gorton (2010), Chapter 2.

illiquid and cannot be traded in a market on short notice, if at all. Such assets would correspond well to the non-marketable asset of our theory. Nevertheless, the ECB regularly provides liquidity to banks against this type of collateral–which includes credit claims and non-marketable retail mortgage-backed debt instruments (RMBDs)–as long as it complies with some eligibility criteria.[11] The use of non-marketable assets pledged at the Eurosystem open market operations as percentage of the total collateral pledged increased steadily with the Subprime and Euro crisis, jumping from about 10% in 2007 to more than 25% in 2012.[12]

We do not explicitly model the frictions that prevents market participants from purchasing, or lending against, non-marketable assets. One reason might be that revealing the information necessary to evaluate the quality of such assets could provide a competitive advantage to the bank's competitors. Providing this information to the central bank, in contrast, is not an issue.

In order to analyze the effect of lending against project by central bank, we make similar assumption as in the previous subsection. The central bank levies a tax with τ at $t = 0$ before banks allocate their resource between cash and securities. At $t = 1$, banks can obtain the cash either by trading their securities on the market or by borrowing from the central bank pledging their project (non-tradable security). We assume that 1) there is no informational asymmetry between banks and the central bank and therefore the quality of project is known to the central bank; and 2) that the central bank lends the amount of debt that banks want under the constraint of its resource τ with net interest rate r and with haircut 0. The central bank chooses the level of tax τ that maximizes the social welfare W. The following proposition describes an equilibrium:

Proposition 11. *Given that $r = 0$, we can have an equilibrium achieving the optimal allocation in which the central bank levies tax with the amount $\tau^* = \lambda$ and banks choose their cash holding level $s^* = 0$ and holds only securities $1 - \lambda$. The market will collapse and the central bank substitutes market but optimal allocation is achieved.*

Proof. First, we characterize the decentralized equilibrium given $\tau = \lambda$. Then we demonstrate that the social welfare given by $\tau^* = \lambda$ and s^* is first-best. Since the central bank levies $\tau = \lambda$, banks know that the central bank has enough resources to lend to all banks with a liquidity shock. Hence, banks would only hold cash to purchase securities. This implies that the (net) expected return on additional cash for banks is always $\frac{1}{p} - \bar{R}$. Since banks do not want to sell securities below their fusubsectionndamental value, the only price at which the market could clear is $p = 1$. However, at that price $\frac{1}{p} - \bar{R} < 0$, and banks do

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[11]These criteria include, notably, that the asset must be denominated in Euro, the debtor must be located in the Euro area, and the debtor's default probability, as defined by the Basel accord, must be lower than 0.4%. Details on non-marketable accepted in guarantee to ECB loans to credit institutions can be found in Sauerzopf (2007) and in Tamura and Tabakis (2013).

[12]See ECB (2013), p. 81.

not want to hold any cash. As a result, given $\tau = \lambda$, market equilibrium is $s^* = 0$. The total output in the economy in this equilibrium is given by

$$W = y + (1 - \lambda) \bar{R},$$

which is the first-best level of welfare. The central bank has no incentive to deviate from $\tau = \lambda$.
∎

8. Conclusion

Our paper proposes a simple theoretical framework to analyze the liquidity choice of banks. Banks require liquidity to meet unexpected shocks and can obtain this liquidity from other banks by selling marketable securities in a market. Interestingly, our theory suggests that banks may hold either too much cash or too little cash, depending on parameter values.

The banks' choice if cash holding deviates from the social optimum when the return on marketable securities is sufficiently variable. In this case, a bank needing liquidity but holding marketable securities with a low return will not be able to obtain as much cash as it would like. When the expected return of securities is sufficiently high, then banks will always hold too little cash, compared to the social optimum. The intuition for this result is that banks do not internalize the effect of their cash holding on the market price of securities. Because of cash-in-the-market pricing, the fact that banks hold too little cash, lowers the price of securities. This, in turn, means that banks cannot continue the long-term project to the extent they would like. A liquidity constraint, such as the LCR, that forces banks to hold enough liquidity can restore the constrained optimum in such situations.

The return on marketable securities is sufficiently variable and the expected return is not too high, then banks will choose to hold too much cash. In this situation, the value of cash for the purpose of continuing the long-term project more than compensate for the fact that cash has a lower "fundamental" return than securities. This can be thought of as a state of crisis, where the market for securities shuts down and banks hoard cash. Central bank intervention can restore the first best in this situation; in particular if the central bank can lend against the value of the non-marketable project.

Appendix

A. Extension: The general case with liquidity shock $x \leq 1$

Until now, we have assumed that bank needs the amount of liquidity 1 to continue its project when it is affected by liquidity shock. In this appendix, we relax this assumption by considering that the amount of liquidity needed to continue project is $x \leq 1$. In this

general case, the assumption (A2) is replaced by

$$\frac{y}{x} > \bar{R} \tag{A2'}$$

First, we determine the date-0 optimal allocation from the point of view of a social planner. The total output of economy becomes

$$W_x(s) = \min\left\{1, \left[1 - \lambda + \lambda(\frac{s+c}{x})\right]\right\} y + \left[(1-\lambda)s - \lambda c\right] + (1-s)\bar{R} \tag{23}$$

The consideration similar to the section 6.1 yields that the optimal level of cash for economy is $s_x^{FB} = \lambda x$ when the social planner has no constraint in redistribution of cash. The constrained-optimal, or second-best, level of cash is denoted by s_x^{SB}. This is given by the following proposition 12:

Proposition 12. *1. When $R_L \geq \frac{(1-\lambda)x}{1-\lambda x}$, the planner is not constrained to redistribution of cash so that $s_x^{SB} = s_x^{FB} = \lambda x$.*

2. When $\hat{R}_x < R_L < \frac{(1-\lambda)x}{1-\lambda x}$, the constrained optimal level of cash s_x^{SB} is $\hat{s}_x$ ($< s_x^{FB}$) where

$$\hat{s}_x \equiv \frac{\lambda\left(\mu x + (1-\mu)R_L\right)}{\lambda\left(\mu + (1-\mu)R_L\right) + 1 - \lambda}, \tag{24}$$

and

$$\hat{R}_x \equiv 1 - \frac{\bar{R} - 1}{\lambda(1-\mu)\left(\frac{y}{x} - 1\right)}. \tag{25}$$

3. When $R_L = \hat{R}_x$, the constrained optimal level of cash s_x^{SB} can be any value between $\left[\hat{s}_x, \frac{x-R_L}{1-R_L}\right]$.

4. When $R_L < \hat{R}_x$, the constrained optimal level of cash s_x^{SB} is $\frac{x-R_L}{1-R_L}$ ($> s_x^{FB}$).

The proof is provided in the appendix F.

The constrained optimal level of cash depends on R_L, the low return for the securities. When R_L is higher than $\frac{(1-\lambda)x}{1-\lambda x}$, the planner can achieve the first-best because the constraint does not bind. When R_L is smaller than $\frac{(1-\lambda)x}{1-\lambda x}$, the planner can no longer transfer the optimal amount of cash. The planner chooses to reduce s for two reasons: First, a reduction of s is necessary to make sure that there is no unused cash after transfers. Second, reducing s allows banks to hold more securities, which can be used to acquire more cash when the return on the securities is low. For these reasons, s^{SB} decreases as R_L decreases until R_L becomes smaller than $\hat{R}$ at which point it jumps to $\frac{x-R_L}{1-R_L}$. The intuition for this result is that R_L becomes so low that the planner prefers that banks hold sufficient cash to continue the project in full scale even when banks hold L-type security. In other words, s^{SB} becomes such that $s + (1-s)R_L = x$. We can easily verify that our main model is a particular case with $x = 1$.

Now, we turn to the decentralized decision on cash holding by individual banks. Consider first aggregate demand and supply of cash in the market. The equilibrium prices and cash holding level satisfy the following lemma 2:

Lemma 2. 1. *Given the cash holding level s chosen at $t = 0$, the equilibrium market return on cash $\frac{1}{p^*}$ always satisfies $1 \leq \frac{1}{p^*} \leq \frac{y}{x}$.*

2. *Given the market price of securities p, the equilibrium cash holding level s^* always satisfies $s^* + (1 - s^*) p R_H \geq x$.*

The proof is similar to that of lemma 1.

Given that $1 \leq \frac{1}{p} \leq \frac{y}{x}$, banks hit by a liquidity shock always prefer to use their cash to continue their project whereas banks that are not affected by liquidity shock always supply their cash in the market. The aggregate supply of cash in the market is

$$S = (1 - \lambda) s$$

The demand for cash depends on whether banks hit by liquidity shock is constrained by the value of their securities or not. Under $s^* + (1 - s^*) p R_H \geq x$, banks with the H-security is never constrained. The aggregate demand depends on the value of L-security. The demand for cash can be written by:

$$D(s) = \begin{cases} \lambda (x - s) & \text{if } x \leq s + (1 - s) p R_L \\ \lambda [\mu (x - s) + (1 - \mu)(1 - s) p R_L] & \text{if } x > s + (1 - s) p R_L \end{cases} \tag{26}$$

Now we consider individual decision on cash holding level by banks. They decide their cash holding level at $t = 0$ maximizing their expected profits. Their decision depends on the expected marginal (net) return on cash conditional on the price of securities p, which is given by

$$\frac{\partial \pi}{\partial s} = \begin{cases} \frac{1}{p} - \bar{R} & \text{if } s + (1 - s) p R_L \geq x \\ \left(\frac{1}{p} - \bar{R}\right) + (1 - \mu) \lambda \left(\frac{y}{x} - \frac{1}{p}\right)(1 - p R_L) & \text{if } s + (1 - s) p R_L < x \end{cases}$$

The equilibrium cash holding level s^* given p must maximize the expected return on cash, therefore must satisfy the following conditions:

- If $s + (1 - s) p R_L \neq x$,
$$\frac{\partial \pi}{\partial s} = 0 \tag{27}$$

- If $s + (1 - s) p R_L = x$
$$\begin{cases} \frac{\partial \pi}{\partial s}(s < s^*) \geq 0 \\ \frac{\partial \pi}{\partial s}(s > s^*) \leq 0 \end{cases} \tag{28}$$

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The conditions in (28) are equivalent to the condition (27) when π is differentiable for all s. In our case, π is piecewise linear in s but the slope changes at s such that $s + (1-s)\, pR_L = x$. Therefore, $\frac{\partial \pi}{\partial s} = 0$ is not sufficient. From this together with the proposition 1, we can characterize market equilibria, represented by the pair of the price p^* and the level s^* of individual cash holding.

Proposition 13. *1. When $\tilde{R}_x < R_L \leq \bar{R}$, there exist an equilibrium with $s^* = \lambda x$ (optimal) where*

$$\tilde{R}_x = \frac{(1-\lambda)\, x}{(1-\lambda x) - (1-\mu)\, \lambda\, (1-x)} \left[\bar{R} - \frac{(1-\mu)\, \lambda\, (1-x)\, y}{(1-\lambda x)\, x} \right] \qquad (29)$$

Equilibrium price is given by

(a) $p^* = \frac{1}{\bar{R}}$ when $\frac{x - \lambda x}{1 - \lambda x} \bar{R} \leq R_L \leq \bar{R}$;
(b) $p^* = \frac{(1-\lambda)x}{1-\lambda x} \frac{1}{R_L}$ when $\tilde{R}_x < R_L < \frac{x - \lambda x}{1 - \lambda x} \bar{R}$.

2. When $\hat{R}_x < R_L < \tilde{R}_x$, there exist an equilibrium with $s^*(< s_x^{SB}) < \lambda x$ and $p^* < 1$. (cash-in-the market pricing) s^* and p^* are determined by The case with small liquidity shock

$$s^* = \frac{\lambda\, (\mu x + (1-\mu)\, p^* R_L)}{\lambda\, (\mu + (1-\mu)\, p^* R_L) + (1-\lambda)} \qquad (30)$$

$$\frac{1}{p^*} = \bar{R} - (1-\mu)\, \lambda \left(\frac{y}{x} - \frac{1}{p^*} \right) (1 - p^* R_L) \qquad (31)$$

3. When $R_L < \hat{R}_x$, there exist an equilibrium with $s^* = \frac{x - R_L}{1 - R_L}$ and $p^* = 1$. (cash hoarding, with the price being the fundamental value)

The proof is provided in the appendix G.

Figure 9 illustrates the equilibrium price and cash level upon the value of security with low quality R_L.
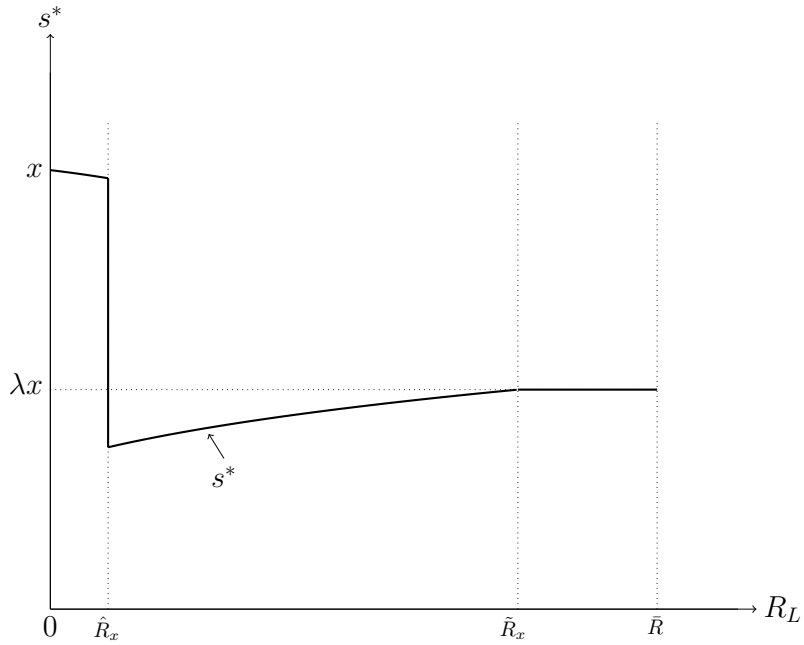

B. Proof of lemma 1

In this appendix, we will demonstrate that equilibrium exists only when $pR_H \geq 1 \geq pR_L$. We prove it by contradiction.

- If $pR_L > 1$, then the bank is not cash-constrained independent of the type of securities, so the marginal return on cash is simply

$$\frac{\partial \pi}{\partial s} = \frac{1}{p} - \bar{R}$$
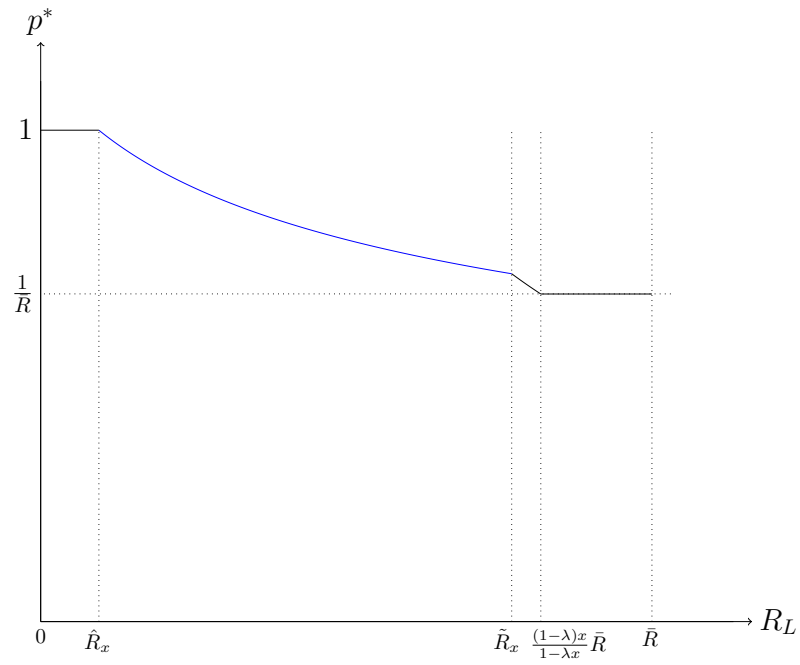
(a) Equilibrium p^*

(b) Equilibrium cash holding level s^*

Figure 9: Equilibrium price and cash holding level in the case with $x \leq 1$

s maximizing individual profit requires $\frac{\partial \pi}{\partial s} = 0$, therefore $p = \frac{1}{\bar{R}}$, which implies $pR_L = \frac{R_L}{\bar{R}} \leq 1$ since $\bar{R} \geq R_L$. This is contradictory with $pR_L > 1$. Therefore, there does not exist an equilibrium with $pR_L > 1$.

- If $pR_H < 1$, then the bank is cash-constrained when it is affected by liquidity shock independent of the type of securities. The marginal return conditional on the type of security θ is

$$\left[\lambda y + (1 - \lambda) \frac{1}{p} \right] - \left[\lambda p R_\theta y + (1 - \lambda) R_\theta \right]$$

yielding the expected marginal return on cash as following:

$$\frac{\partial \pi}{\partial s} = \left(\frac{1}{p} - \bar{R} \right) + \lambda \left(y - \frac{1}{p} \right) (1 - p\bar{R})$$

We know that $p\bar{R} < 1$ since $p\bar{R} \leq pR_H < 1$. From lemma 1, $\frac{1}{p} \leq y$ in equilibrium. As a consequence, $\frac{\partial \pi}{\partial s} > 0$. This is contradictory with $\frac{\partial \pi}{\partial s} = 0$ required by profit maximizing s. Therefore, there does not exist an equilibrium with $pR_H < 1$.

C. Proof of proposition 2

For the convenience, we consider the two following cases separately: i) equilibria with $S = D$ and ii) equilibria with $S > D$.

1. Equilibrium with $S = D$

$D = S$ gives

$$s^* = \frac{\lambda (\mu + (1 - \mu) pR_L)}{\lambda (\mu + (1 - \mu) pR_L) + (1 - \lambda)} \tag{32}$$

We consider two subcases:

Equilibrium p^* and s^* are determined by the individual profit maximization yielding $\frac{\partial \pi}{\partial s} = 0$ and market equilibrium condition $S = D$.

i) When $pR_L = 1$:

The equation (32) gives

$$s^* = \lambda$$

and the marginal return on cash is given by

$$\frac{\partial \pi}{\partial s} = \frac{1}{p} - \bar{R}$$

s maximizing individual profit requires $\frac{\partial \pi}{\partial s} = 0$, which yields

$$p = \frac{1}{\bar{R}}.$$

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Combining is expression with $pR_L = 1$ implies $\frac{R_L}{\bar{R}} = 1$, which can only hold if $R_H = R_L = \bar{R}$.

We can conclude that there exist an equilibrium with $s^* = \lambda$ and $p^* = \frac{1}{\bar{R}}$ when $R_L = R_H$.

ii) When $pR_H \geq 1 > pR_L$:

s maximizing individual profit requires $\frac{\partial \pi}{\partial s} = 0$, which yields

$$\frac{1}{p^*} = \bar{R} - (1-\mu)\lambda\left(y - \frac{1}{p^*}\right)(1 - p^* R_L) \tag{33}$$

Since $p^* \leq 1$, we have

$$R_L \geq 1 - \frac{\mu(R_H - 1)}{(1-\mu)(\lambda y + 1 - \lambda)} = \hat{R}$$

Equilibrium s^* and p^* are determined by (32) and (33) under the condition that p^* satisfies $pR_H \geq 1 > pR_L$. p^* determined by (33) satisfies $pR_H \geq 1 > pR_L$ whenever $\hat{R} \leq R_L < \bar{R}$. We can easily check that s^* is lower than λ from (32) when $\hat{R} \leq R_L < \bar{R}$. From (32) and (22), we can check that s^* is lower than $\hat{s}$ under $p^* < 1$, which implies that $s^* < \hat{s}$ when $R_L > \hat{R}$.

2. Equilibria with $S > D$

In such equilibrium, if exist, $p^* = 1$ from proposition 1. Since $R_H > 1$ by definition, the case that we consider $pR_H \geq 1 \geq pR_L$ becomes $R_L \leq 1 < R_H$. We consider two subcases:

i) When $R_L = 1$:

In such case, the condition (32) gives $s^* = \lambda$. However, the marginal return on cash is $\frac{\partial \pi}{\partial s} = 1 - \bar{R} < 0$, which implies that $s = 0$ is profitable deviation. Therefore, there does not exist an equilibrium with $R_L = 1$ and $S > D$.

ii) When $R_L < 1$:

In this case, we have always $s + (1-s)R_L \leq 1$. We consider separately the case with $s + (1-s)R_L < 1$ and that with $s + (1-s)R_L = 1$:

When $s + (1-s)R_L < 1$: This always holds under $R_L < 1$ except at $s = 1$. On the one hand, no profitable deviation condition gives

$$\frac{\partial \pi}{\partial s} = 1 - \bar{R} + (1-\mu)\lambda(y - 1)(1 - R_L) = 0$$

yielding

$$R_L = 1 - \frac{\mu(R_H - 1)}{(1-\mu)(\lambda y + 1 - \lambda)} = \hat{R}$$

On the other hand, from $S > D$,

$$s^* > \frac{\lambda\left(\mu + (1-\mu)R_L\right)}{\lambda\left(\mu + (1-\mu)R_L\right) + 1 - \lambda} = \hat{s}$$

We can conclude that when $R_L = \hat{R}$ given $\hat{R} \geq 0$, a pair of p and s such that $p = 1$ and $s \in (\hat{s}, 1)$ can be an equilibrium. If $\hat{R} < 0$, there does not exist such an equilibrium.

When $s + (1-s)R_L = 1$: Given $R_L < 1$, the only case satisfying the above is $s^* = 1$ and

$$\frac{\partial \pi}{\partial s}(s < 1) = 1 - \bar{R} + (1-\mu)\lambda(y-1)(1-R_L) \geq 0$$

should hold in order for $s = 1$ to be an equilibrium.

We can conclude that if $R_L < \hat{R}$ given that $\hat{R} > 0$, $p^* = 1$ and $s^* = 1$ is an equilibrium. If $\hat{R} \leq 0$, there does not exist such an equilibrium.

D. Proof of proposition 3

From equation (11),

$$R_L = \bar{R}\left(1 - c_v\sqrt{\frac{\mu}{1-\mu}}\right) \tag{34}$$

Characterizing equilibrium in a way similar to proposition 2 by substituting (34) for R_L in expected marginal return on cash (7) yields the following: i) when c_v is higher than $\hat{c}_v$, equilibrium cash level is always 1 ; and ii) when c_v is lower $\hat{c}_v$, equilibrium cash level is determined by

$$\frac{1}{p^*} - \bar{R} + (1-\mu)\lambda\left(y - \frac{1}{p^*}\right)(1-K) = 0 \tag{35}$$

$$s^* = \frac{\lambda[\mu + (1-\mu)K]}{\lambda[\mu + (1-\mu)K] + (1-\lambda)} \tag{36}$$

where $K \equiv p^*\bar{R}\left(1 - c_v\sqrt{\frac{\mu}{1-\mu}}\right)$. Note that $0 \leq K \leq 1$. ($\because K = p^*R_L \leq 1$ in equilibrium by lemma 1. $c_v \leq \sqrt{\frac{1-\mu}{\mu}}$ yields $K \geq 0$.)

In what follows, we prove $\frac{\partial s^*}{\partial c_v} < 0$. By chain rule, $\frac{\partial s^*}{\partial c_v} = \frac{\partial K}{\partial c_v}\frac{\partial s^*}{\partial K}$.

$$\frac{\partial s^*}{\partial K} = \frac{\lambda(1-\lambda)(1-\mu)}{[\lambda[\mu + (1-\mu)K] + (1-\lambda)]^2} > 0 \tag{37}$$

Now, we check the sign of $\frac{\partial K}{\partial c_v}$. Deriving K w.r.t. c_v yields

$$\frac{\partial K}{\partial c_v} = \left[\left(1 - c_v\sqrt{\frac{\mu}{1-\mu}}\right)\frac{\partial p^*}{\partial c_v} - p^*\sqrt{\frac{\mu}{1-\mu}}\right]\bar{R} \tag{38}$$

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By deriving equation (35) w.r.t. c_v given $\bar{R}$, λ, μ and y constant, we can obtain

$$\frac{\partial p^*}{\partial c_v} = \frac{(1-\mu)\lambda\left(y - \frac{1}{p^*}\right)K}{\frac{1}{p^{*2}} + (1-\mu)\lambda\left(y - \frac{1}{p^*}\right)K} \tag{39}$$

Plugging (39) into (38) yields

$$\frac{\partial K}{\partial c_v} = -\frac{\frac{1}{p^*}\sqrt{\frac{\mu}{1-\mu}}}{\frac{1}{p^{*2}} + (1-\mu)\lambda\left(y - \frac{1}{p^*}\right)K} < 0 \tag{40}$$

From (37) and (40), $\frac{\partial s^*}{\partial c_v} = \frac{\partial K}{\partial c_v}\frac{\partial s^*}{\partial K} < 0$.

E. Proof of proposition 6

Characterization of equilibrium similar to proposition 6 allow to obtain proposition 6.1 and 6.2 (b). 6.2 (c) is straightforward from (15). It remains us to demonstrate 6.2(a). By chain rule, $\frac{\partial s^*}{\partial \bar{R}} = \frac{\partial K}{\partial \bar{R}}\frac{\partial s^*}{\partial K}$. Deriving K w.r.t. $\bar{R}$ yields

$$\frac{\partial K}{\partial \bar{R}} = \left(1 - c_v\sqrt{\frac{\mu}{1-\mu}}\right)\left(\frac{\partial p^*}{\partial \bar{R}}\bar{R} + p^*\right)$$

Since $\frac{\partial s^*}{\partial K} > 0$ from (37) and $c_v\sqrt{\frac{\mu}{1-\mu}} \leq 1$ ($\because c_v \leq \sqrt{\frac{1-\mu}{\mu}}$), the sign of $\frac{\partial s^*}{\partial \bar{R}}$ only depends upon $\frac{\partial p^*}{\partial \bar{R}}\bar{R} + p^*$. Deriving (35) w.r.t. $\bar{R}$ yields

$$\frac{\partial p^*}{\partial \bar{R}} = \frac{1 + p^*\left(y - \frac{1}{p^*}\right)\left(1 - c_v\sqrt{\frac{\mu}{1-\mu}}\right)}{\frac{1}{p^{*2}}\{1 - (1-\mu)\lambda\} + \left(1 - c_v\sqrt{\frac{\mu}{1-\mu}}\right)\left[\frac{1}{p^{*2}}(1-\mu)\lambda + \left(y - \frac{1}{p^*}\right)\bar{R}\right]}$$

Plugging this into $\frac{\partial p^*}{\partial \bar{R}}\bar{R} + p^*$, we obtain

$$\frac{\partial p^*}{\partial \bar{R}}\bar{R} + p^* = \frac{\left(\frac{1}{p^*} - \bar{R}\right) - \frac{1}{p^*}(1-\mu)\lambda\sqrt{\frac{\mu}{1-\mu}}}{\frac{1}{p^{*2}}\{1 - (1-\mu)\lambda\} + \left(1 - c_v\sqrt{\frac{\mu}{1-\mu}}\right)\left[\frac{1}{p^{*2}}(1-\mu)\lambda + \left(y - \frac{1}{p^*}\right)\bar{R}\right]} \tag{41}$$

From (35), $\frac{1}{p^*} - \bar{R}$ is always negative in equilibrium, in turn, $\frac{\partial p^*}{\partial \bar{R}}\bar{R} + p^*$ is always negative. As a result, $\frac{\partial s^*}{\partial \bar{R}} < 0$. Thus, if $\bar{R} > \hat{\bar{R}}$, the relationship between s^* and $\bar{R}$ is negative.

F. Proof of proposition 12

When $R_L \geq x$, social planner is never constrained by the value of securities when it transfers cash. The problem is same as that of unconstrained social planner. $s_x^{SB} = s_x^{FB} =$

λx. Now we turn to the case with $R_L < x$. In such case, The aggregate amount of redistributed cash is given by

$$C_x = \begin{cases} (1 - \lambda) s & \text{if} \quad s + (1 - s) R_L \geq x, \\ \lambda \left[\mu (x - s) + (1 - \mu) (1 - s) R_L \right] & \text{if} \quad s + (1 - s) R_L < x. \end{cases} \tag{42}$$

When $s + (1 - s) R_L \geq x$, the constrained social planner can attain first-best. Now, consider the case with $s + (1 - s) R_L < x$. Let $\hat{s}_x$ denote the level of s such that there is no cash left over after transfers. It is the value of s that solves $\lambda \left[\mu (x - s) + (1 - \mu) (1 - s) R_L \right] = (1 - \lambda) s$. It is easy to verify that $\frac{\partial W}{\partial s} > 0$ if $s < \hat{s}_x$. Indeed, when $s < \hat{s}_x$ an additional unit of cash can be used to continue a larger share of the project without resulting in any leftover cash after transfers. When $s \geq \hat{s}_x$, we have

$$\frac{\partial W}{\partial s} = (1 - \lambda) + \lambda \mu + \lambda (1 - \mu) \left(\frac{y}{x} - \left(\frac{y}{x} - 1 \right) R_L \right) - \bar{R}$$

The sign of this expression depends on R_L, but not on s. Let $\hat{R}_x$ denote the value of R_L such that $\frac{\partial W}{\partial s} = 0$. If $R_L > \hat{R}_x$, then $\partial W / \partial s < 0$ and if $R_L < \hat{R}$, then $\partial W / \partial s > 0$. From the above consideration, the constrained optimal level of cash when $R_L < 1$ depends on the two threshold levels of s: s such that $s + (1 - s) R_L = x$, or $s = \frac{x - R_L}{1 - R_L}$ on the one hand and $\hat{s}_x$ on the other hand. Two following cases should be considered separately:

1. When $\frac{x - R_L}{1 - R_L} < \hat{s}_x$, or $R_L > \frac{(1 - \lambda) x}{1 - \lambda x}$:

   Note that in such case, $\frac{x - R_L}{1 - R_L} < \lambda x < \hat{s}_x$: i) When $s < \frac{x - R_L}{1 - R_L}$, $\frac{\partial W}{\partial s} > 0$ since $s < \frac{x - R_L}{1 - R_L}$ and $s < \hat{s}_x$; ii) When $\frac{x - R_L}{1 - R_L} < s < \lambda x$, $\frac{\partial W}{\partial s} > 0$; iii) When $s > \lambda x > \frac{x - R_L}{1 - R_L}$, $\frac{\partial W}{\partial s} < 0$. Welfare maximizing cash holding level under the constraint is therefore $s_x^{SB} = \lambda x$ which equal to the first-best level.

2. When $\frac{x - R_L}{1 - R_L} > \hat{s}_x$, or $R_L < \frac{(1 - \lambda) x}{1 - \lambda x}$:

   Note that in such case, $\hat{s}_x < \lambda x < \frac{x - R_L}{1 - R_L}$: i) When $s < \hat{s}_x$, $\frac{\partial W}{\partial s} > 0$. ii) When $s > \hat{s}_x$, the sign of $\frac{\partial W}{\partial s}$ depends on R_L. If $R_L > \hat{R}_x$, then $\frac{\partial W}{\partial s} < 0$. Therefore, $s_x^{SB} = \hat{s}_x$. If $R_L < \hat{R}$, then $\frac{\partial W}{\partial s}$ is positive as far as $s > \frac{x - R_L}{1 - R_L}$ and becomes negative when $s < \frac{x - R_L}{1 - R_L}$. Therefore, $s_x^{SB} = \frac{x - R_L}{1 - R_L}$.

   We can conclude that i) when $R_L > \frac{(1 - \lambda) x}{1 - \lambda x}$, $s_x^{SB} = s_x^{FB} = \lambda x$; ii) when $\hat{R}_x < R_L < \frac{(1 - \lambda) x}{1 - \lambda x}$, $s^{SB} = \hat{s}_x < s_x^{FB}$; iii) when $R_L < \hat{R}_x$, $s^{SB} = \frac{x - R_L}{1 - R_L} > s_x^{FB}$.

G. Proof of proposition 13

For the convenience, we consider the two following cases separately: i) equilibria with $S = D$ and ii) equilibria with $S > D$.

1. Equilibrium with $S = D$

Equilibrium p^* is determined by equilibrium condition (27) and / or (28).
We consider two subcases:

i) When $s + (1 - s) pR_L \geq x$:

$S = D$ gives

$$s^* = \lambda x$$

and the marginal return on cash is given by

$$\frac{\partial \pi}{\partial s} = \frac{1}{p} - \bar{R}$$

a) Equilibrium condition (27) yields

$$p^* = \frac{1}{\bar{R}}.$$

Combining this expression with $s + (1 - s) pR_L \geq x$ implies

$$R_L \geq \frac{x - \lambda x}{1 - \lambda x} \bar{R}$$

We can conclude that there exist an equilibrium with $s^* = \lambda x$ and $p^* = \frac{1}{\bar{R}}$ when $R_L \geq \frac{x-\lambda x}{1-\lambda x}\bar{R}$.

b) Equilibrium condition (28) is relevant only when $s + (1 - s) pR_L = x$. Plugging $s^* = \lambda x$ into $s + (1 - s) pR_L = x$ yields

$$p^* = \frac{x - \lambda x}{1 - \lambda x} \frac{1}{R_L}$$

Equilibrium condition (28) yields

$$\frac{\partial \pi}{\partial s} (s < s^*) = \left(\frac{1}{p} - \bar{R} \right) + (1 - \mu) \lambda \left(\frac{y}{x} - \frac{1}{p} \right) (1 - pR_L) \geq 0$$

and

$$\frac{\partial \pi}{\partial s} (s > s^*) = \frac{1}{p} - \bar{R} \leq 0$$

Plugging p^* into the above two conditions, they yield respectively $R_L \geq \tilde{R}_x$ given by (29) and $R_L \leq \frac{x-\lambda x}{1-\lambda x}\bar{R}$.

We can conclude that $s^* = \lambda x$ and $p^* = \frac{x-\lambda x}{1-\lambda x}\frac{1}{R_L}$ is an equilibrium when $\tilde{R}_x \leq R_L \leq \frac{x-\lambda x}{1-\lambda x}\bar{R}$.

ii) When $s + (1 - s) \, pR_L < x$:

$D = S$ yields s^* given by (30). Note that given $s + (1 - s) \, pR_L < x$, $s^* < \lambda x$. The individual incentive condition for equilibrium $\frac{\partial \pi}{\partial s} = 0$ yields p^* given by (31). Since $p^* \leq 1$ should hold, we have

$$R_L \geq \hat{R}_x$$

The upper bound of R_L is given by $s + (1 - s) \, pR_L = x$ and $s^* = \lambda x$ as well as (31) , which yields

$$R_L < \tilde{R}_x$$

We can therefore conclude that the pair of s^* and p^* given by (30) and (31) is an equilibrium when $\hat{R}_x \leq R_L < \tilde{R}_x$.

2. Equilibria with $S > D$

In such equilibrium, if exist, $p^* = 1$, implying that the price of the security is its fundamental value since there exist excess supply in the market. We consider three subcases:

i) When $s + (1 - s) \, R_L > x$:

In such case, $S > D$ requires $s^* > \lambda x$. However, the marginal return on cash is $\frac{\partial \pi}{\partial s} = 1 - \bar{R} < 0$, which implies that $s = 0$ is profitable deviation. Therefore, there does not exist an equilibrium with $s + (1 - s) \, R_L \geq x$ and $S > D$.

ii) When $s + (1 - s) \, R_L = x$:

$s + (1 - s) \, R_L = x$ gives

$$s^* = \frac{x - R_L}{1 - R_L}$$

$S > D$ requires $s^* > \lambda x$. This yields

$$R_L < \frac{x - \lambda x}{1 - \lambda x}$$

We know that $\frac{\partial \pi}{\partial s} = 1 - \bar{R} < 0$ at s such that $s + (1 - s) \, R_L = x$. Therefore, the equilibrium condition (28) requires:

$$\frac{\partial \pi}{\partial s} (s < s^*) = \left(1 - \bar{R}\right) + (1 - \mu) \, \lambda \left(\frac{y}{x} - 1\right) (1 - R_L) \geq 0$$

because $\frac{\partial \pi}{\partial s} (s > s^*) = 1 - \bar{R}$ is always negative. This yields

$$R_L < \hat{R}_x$$

Note that $\hat{R} < \frac{x - \lambda x}{1 - \lambda x}$. We can therefore conclude that $p^* = 1$ and $s^* = \frac{x - R_L}{1 - R_L}$ is an equilibrium when $R_L < \hat{R}$ given that $\hat{R} > 0$. If $\hat{R} \leq 0$, there does not exist such an equilibrium. Note that as R_L becomes smaller, s^* becomes bigger.

iii) When $s + (1 - s) R_L < x$:

On the one hand, no profitable deviation condition gives

$$\frac{\partial \pi}{\partial s} = 1 - \bar{R} + (1 - \mu) \lambda (y - 1) (1 - R_L) = 0$$

yielding

$$R_L = \hat{R}_x$$

On the other hand, from $S > D$,

$$s^* > \frac{\lambda (\mu + (1 - \mu) R_L)}{\lambda (\mu + (1 - \mu) R_L) + 1 - \lambda} = \hat{s}_x$$

We can conclude that a pair of $p^* = 1$ and any value of $s \in \left(\hat{s}_x, \frac{x - \hat{R}_x}{1 - \hat{R}_x} \right)$ can be an equilibrium when $R_L = \hat{R}$ given $\hat{R} > 0$, . If $\hat{R} \leq 0$, there does not exist such an equilibrium.

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