The pitfalls of central bank independence when agents are learning

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Abstract

Common wisdom holds that the central banker has to be independent from the government under the hypothesis of rational expectations. However, the assumption of learning puts into question the optimal combinations of conservatism and independence the government should choose. We show in this paper using a New Keynesian model that the central banker should be hawkish as learning, instrument- and goal-independence increase. For a substantial level of instrument independence, an increase in learning and goal-independence reduce the inflation bias and the stabilization bias, however the welfare loss simultaneously increases.

Keywords: Central bank independence, conservatism, negotiation, adaptive learning, optimal monetary policy, inflation contract.

JEL Classification: C63, D83, E52, E58, .

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1 Introduction

Central bank (CB) independence has been a major trend in central banking across the world since the 1990’s. However, “independence does not mean isolation, which is why it is important for a dialogue between the central bank and the democratically elected institutions, as well as directly with the public at large, to be maintained” as Yves Mersch, Member of the ECB’s Executive board, underlines it in its speech at the “Symposium on Building the Financial System of the 21st Century”, in March 2017. It is widely acknowledged from literature on CB independence that central bankers should be independent from the government to shield them from political pressures such that they remain focused on the objective of maintaining price stability. The severity of the recent financial and economic crisis revives the questioning of this commonly-shared view.

The independence of the CB can be decomposed in two components, i.e., the goal independence and the instrument independence (Debelle and Fischer 1994). First, the guarantee of being goal-independent is characterized by the CB that is free from political pressures when defining its policy objectives and preferences. Second, the instrument-independence can be observed by the way the CB freely adjusts its policy tools targeting policy goals.\footnote{For example, the Federal Reserve and the ECB are fully independent but the Bank of England and the Bank of New Zealand are lacking only goal independence.}

The literature on CB independence has profusely stated the negative relationship between inflation and the CB independence for both types of independence. Following Kydland and Prescott (1977), McCallum (1997) and Mishkin (2017) among others, in the absence of independence and under discretionary monetary policy, a higher inflationary bias appears compared to commitment due to the time-inconsistency issue. Furthermore, under rational expectations (RE), a higher level of instrument-independence
reduces inflation expectations if the CB is conservative according to Lippi (2000) but at the cost of increasing the stabilization bias. The latter is a dynamic phenomenon, and is reflected by a higher volatility of inflation out of its steady-state level under discretionary policy, compared to the case where the CB conducts its policy under commitment. Following the literature, the level of CB independence must remain high to keep monetary policy effective and not be affected by electoral cycle as argue Alesina and Stella (2011), when the tasks are technical in nature and monitoring their quality happens to be complex, then the delegation of decision-making authority to non-elected bureaucrats with career concerns is especially beneficial. Yet, the relation between the optimal level of instrument-independence and conservatism remains widely discussed. Indeed, the optimal level of instrument-independence can adjust with the degree of conservatism, since Hugues Hallett and Weymark (2005) and, Eijffinger and Hoeberichts (2008) stress that decreasing the level of instrument-independence should be balanced by a higher degree of conservatism which allows a continuum of possible optimal policies under RE with different combinations of independence level and conservatism. Lately, Hefeker and Zimmer (2011) demonstrated that more instrument-independence should be supported with more conservatism under RE when there is uncertainty about the CB’s output-gap target. However, a CB with a high level of instrument-independence cannot and should not control all main macroeconomic variables. As Orphanides (2013) has emphasized, the CB should not be overburdened by too many objectives (financial and fiscal stabilization, full employment...) since losing full independence and credibility are at stake, i.e., reducing the CB’s ability to preserve price stability and to contribute to crisis management.

The literature considering the relationship between conservatism and instrument-independence assumes that private agents, i.e., households and firms, are forming rational expectations (RE) and states that conservatism and instrument-independence
are complementary. The hypothesis of RE is however to be reconsidered in the light of the recent financial crisis. Indeed, the fact that private agents are frequently characterized by biased expectation formation (Hollmayr and Kühl 2016), was stressed during the last economic meltdown. However private expectations represent a considerable channel for the transmission of monetary policy. For that matter the central banker that aims at stabilizing inflation must anchor private agents’ expectations. To model and characterize the formation of such biased expectations, a growing theoretical and empirical literature has emphasized the bounded rationality of private agents including the seminal work of Simon (1991). A further feature of bounded rationality is the fact that private agents are learning over time the way their economic environment, and can be modelled as Evans and Honkapohja (2001, 2003) have shown. Notice that learning algorithm also helps selecting among multiple RE equilibria (Evans and Honkapohja 1994), and is therefore justified since the combination of conservatism and instrument-independence yields many optimal solutions. Learning, as a departure from RE, raises the risk for a possible economic instability that calls the central banker to anchor inflation expectations and pushes for a more aggressive response of the interest rate policy to expected inflation under learning compared to the case under RE (among others, Ferrero 2007 and Airaudo et al. 2015).

The aim of this paper is to study how optimal monetary policy should be set and to examine the optimal combination of CB independence and conservatism, compared to the RE equilibrium when agents are learning. We focus on the one hand how the level of the CB independence influences the optimal monetary policy under discretion and affects the economic dynamics and, takes of the inflation aversion of the government and the CB. The model integrating the issue of independence via a parameter is based upon Bernanke and Mishkin (1997) that describe the conduct of an independent monetary policy, and includes a conservative central banker. The difference between
the inflation aversions of the CB and the government, also called conservatism, repre-
sents goal independence, and the negotiation level stands for instrument independence. 
According to its level of inflation aversion, the CB can be qualified as being conserv-
ative or hawkish in the terminology of Rogoff (1985). More precisely, conservatism 
is defined as the CB’s weight on inflation that exceeds the weigh on inflation set by 
the elected government, i.e., the CB prefers maintaining a low and stable inflation rate 
compared to the government. The main contributions of this paper are: (1) under 
learning, full independence is not optimal since less instrument-independence improves 
the social welfare; (2) an increase in learning considerably raises social loss but also 
partially offsets the stabilization bias as the CB is more goal-independent; (3) a higher 
CB independence of instrument and goals leads the central banker to be hawkish but 
an increase in learning reduces this trend; (4) a linear inflation contract under learning 
can either yield a liberal or conservative central banker to offset the stabilization bias 
whereas under RE, the central banker should be liberal.

The remainder of the paper is organized as follows. Section 2 presents the model 
and its features. The next section solves the benchmark model under RE. Section 4 
shows the equilibrium solutions when agents are learning with constant gain. Section 
5 studies how learning with decreasing gain leads to similar results. Section 6 solves 
for the optimal combination of independence and conservatism. Section 7 examines the 
institutional design combining CB independence and inflation contract. Finally, the 
last section concludes.
2 The model

2.1 The baseline equations

The economic environment is described by a standard micro-based New Keynesian model, which incorporates optimizing private sector behavior and nominal rigidities, and is widely used in the recent monetary policy literature (Clarida, Galí and Gertler 1999).

The NK model is based on two baseline equations, the Phillips curve and, the aggregate demand or IS equation that is derived from the optimal consumption decision of a representative household. The Phillips curve, or forward-looking inflation adjustment equation, is:

\[ \pi_t = \beta E_t^* \pi_{t+1} + \kappa x_t + e_t, \]  

(1)

where \( e_t \sim N(0, \sigma_e) \) is the cost-push shock, \( \pi_t \) the inflation rate, \( \beta \in (0, 1) \) the discount factor for private agents, \( E_t^* \pi_{t+1} \) the inflation expectations in period \( t \) for period \( t + 1 \) where the expectations operator \( E_t^* \) embodies private agents’ expectations conditional on information set available at time \( t \), with the asterisk highlighting that private agents may form either RE or expectations using learning. \( \kappa \) is a composite parameter defined as \( \kappa = \frac{(1-\vartheta)(1-\vartheta\beta)}{\vartheta}(1+\varphi) \), with \( \varphi \) standing for the inverse of the steady-state elasticity of labor supply and \( \vartheta \) the share of firms that do not optimally adjust but simply update in period \( t \) their previous price by the steady-state inflation rate. The composite parameter \( \kappa \) is the output-gap elasticity of inflation and captures the effects of the output gap on real marginal costs and thus on inflation. Finally, \( x_t \) represents the output-gap level.

The New Keynesian IS equation is given by

\[ x_t = E_t^* x_{t+1} - \sigma^{-1}(r_t - E_t^* \pi_{t+1}), \]  

(2)
where \( r_t \) is the nominal short-term interest rate and \( \sigma \) the risk aversion of households. To simplify the analysis, we assume there is no demand shock and misspecification in the IS equation since the CB can neutralize shocks affecting the aggregate demand by optimally setting the interest rate, which is the policy instrument here. Underpinnings to (1) are that, in a monopolistically competition environment, each firm derives its price-setting decision from an explicit optimization problem. More precisely, whenever possible, the firm sets the nominal price of its product to maximize profits subject to the constraint on the frequency of future price adjustments as defined in Calvo (1983).

### 2.2 Institutional design and policy objectives

One of the most problematic issues in discretionary monetary policy is that the government chooses an output-gap target above its potential level, which produces an inflation bias. The inflation bias is closely linked to the discretionary nature of monetary policy and its inherent time-inconsistency problem, and is translated into a steady-state inflation rate higher than the inflation target rate. Without further assumption on the CB institutional design, the CB shares the government’s objective function and would set a discretionary policy, thus the CB could not avoid the time-inconsistency problem and the associated inflation bias. The question of independence is therefore a key issue.

The government and the CB have their own preference for stabilizing the inflation. The degree of relative conservatism of the CB compared to the government, is denoted by the exogenous parameter \( \phi = \delta^{CB} - \delta^{G} \) where \( \phi \) represents the difference between \( \delta^{CB} \) that is the inflation aversion for the CB and, \( \delta^{G} \) the endogenous inflation aversion for the government. If \( \phi = 0 \), i.e., \( \delta^{CB} = \delta^{G} \), then the CB is fully liberal and shares the goal of the government concerning the stabilization of the inflation level. It thus means that the CB lacks of goal independence. Inversely, if \( \phi = 1 \), i.e., \( \delta^{CB} = 1 + \delta^{G} \), the CB is Rogoff-conservative relative to the behaviour of the government, and is entirely
goal-independent. It is here crucial to make the difference between inflation aversion and relative conservatism. It is indeed difficult to draw one particular degree of inflation aversion for the government since it varies across countries.\footnote{We can take the example of Germany where the inflation aversion for both the CB and the government is high, thus leading to $\phi = 0$. In this case, the CB is certainly liberal but cares very much about inflation level (Lippi, 2000).}

In this paper, we assume that the government faces a high level of unemployment and in a political agenda, will prefer stabilizing the output gap rather than the inflation, meaning that its inflation aversion, $\delta^G$, is very low and close to 0. We will yet study what occurs as $\delta^G$ varies from 0 to 1, which will condition the degree of relative conservatism $\phi$. In the case where the CB is goal-dependent, then the latter can receive political pressures from elected officials that are often motivated by short-run electoral considerations. As a result, the CB would discard the longer-run inflationary consequences of expansionary policies.

The government shares the same expected loss function with the society and it is assumed to take the standard form:

$$L_t^G = \frac{1}{2} E_t \sum_{i=0}^{+\infty} \beta^i [\delta^G \pi^2_{t+i} + \alpha (x_{t+i} - \tilde{x})^2],$$

(3)

where $\tilde{x}$ stands for the overambitious output-gap target and the inflation target is set to zero for simplicity and $\alpha > 0$ the relative weight assigned to the objective of output-gap stabilization.

Under RE, if the appointed central banker was conservative, i.e., a higher preference to achieve the stabilization of price level may reduce or even offset the inflation bias and the stabilization bias. Social loss is a function of the variance of both inflation and the output gap.\footnote{The loss function (3) could be micro-founded by deriving the utility function of representative agent as in Giannoni and Woodford (2003).} The overly ambitious output target, which is common in the Barro-
Gordon framework, is also present in the formulation given in (3). Thus, discretionary monetary policy set to minimize social loss (3) would induce an average inflation bias and, in addition a stabilization bias.

The central bank implements discretionary monetary policy to minimize the conditional expectation of the loss function:\footnote{Issues of learning when monetary policy is under commitment have been studied by Evans and Honkapohja (2006), and Mele, Molnár and Santoro (2014). The first study shows that both rational expectations commitment equilibrium (RECE) and rational expectations discretionary equilibrium (REDE) are attainable, while the second suggests that the optimal monetary policy drives the economy far from the RECE and to the REDE.}

\[
L^{CB}_t = \frac{1}{2} ET \sum_{i=0}^{+\infty} \beta^i [\delta^{CB} \pi^2_{t+i} + \alpha (x_{t+i} - \bar{x})^2],
\]

In Hefeker and Zimmer (2011), the CB sets the output-gap target that is not disclosed to the government when conducting the policy.\footnote{An alternative modelling of the output-gap target is to set it stochastic. The stochastic part of the CB’s preferences corresponds to the way the structure of the CB aggregates heterogeneous social preferences (Faust and Svensson 2002 and Jensen 2002). Shifts in the CB’s output-gap target can be explained by the appointment of a new central banker. Alternatively, as Westelius (2009) suggests, assuming that the stochastic nature of the CB’s output-gap target could reflect measurement errors of the potential output.} The feature of their paper based on Lippi (2000) allows to clearly distinguishing the causes of low inflation, by separating inflation aversion (goal independence) from negotiation (instrument independence) for the CB. We assume that the CB (the Agent) can negotiate with the government (the Principal) to conduct its policy and choose its level of instrument-independence, \( \mu \), from the government and we exclude any external political interference.

The government can interfere with monetary policy making and this intervention is embodied in the objective function that is obtained thanks to a weighted average of the central bank and the government’s objectives. We therefore consider a Stackelberg game where both players, the government and the CB, share the same policy objective with different inflation aversions. Thus, the government first sets \( \mu \) and \( \phi \) as the leader.
The central bank (CB) as the follower conducts policy under discretion taking account of the interference of the government through minimizing the following lost function:

\[
L^P = \mu L^C_{t} + (1 - \mu) L^G_{t}
\]

\[
= \frac{1}{2} E_t \sum_{i=0}^{+\infty} \beta^i \left\{ (\mu \phi + \delta^G) \pi^2_{t+i} + \alpha (x_{t+i} - \bar{x})^2 \right\},
\]

(5)

where \(0 \leq \mu \leq 1\) measures the level of negotiation of the central bank with the government regarding its preferences between stabilizing inflation or the output gap. Thus, \(\mu\) represents the level of instrument independence of the CB, or as in Lippi (2000), the probability that the CB conducts its policy at his/her discretion. The full instrument-dependence of the CB towards the government is obtained if \(\mu = 0\) and conversely, when \(\mu = 1\), the CB is fully independent.

### 2.3 The learning algorithm

We follow Molnár and Santoro (2014) by considering both constant and decreasing-gain algorithm. We first assume that the learning gain is constant (section 4.1), i.e., that \(\gamma = \gamma_t = \gamma_{t+1} \in (0, 1)\), so that the model remains tractable and allows us to obtain analytical results.

We then relax (section 4.2) this restrictive assumption and examine the equilibrium solutions with decreasing gain, i.e., \(\gamma_t = \frac{1}{t}\). The decreasing gain equals 1 when \(t = 1\) and tends to 0 as \(t \to \infty\), thus the learning coefficient is also bounded with \(\gamma_t \in (0, 1)\). The interest of considering decreasing gain lies in the fact that as time goes by, private agents are more skilled with learning process and are hence increasingly confident in their expectations. The fact that the learning gain decreases from 1 to 0 over time yields that the economy jumps from one dynamic path with higher constant learning gain to another one with lower constant learning gain (Molnár and Santoro 2014, André
and Dai 2017).

The learning assumption is based on the observation that private agents have limited rationality, corresponding to a constrained knowledge of the process ruling the way endogenous variables evolve. To forecast the evolution of those variables, private agents recursively estimate a Perceived Law of Motion (PLM) in the terminology of Evans and Honkapohja (2001), based on the previous values of inflation and the output gap. This manner of forecasting is consistent with the law of motion that the CB follows under RE, by using the following deterministic learning algorithm:

\[
E_t \pi_{t+1} \equiv a_t = a_{t-1} + \gamma_t (\pi_{t-1} - a_{t-1}),
\]

(6)

\[
E_t x_{t+1} \equiv b_t = b_{t-1} + \gamma_t (x_{t-1} - b_{t-1}).
\]

(7)

3 The benchmark equilibrium under RE

This section provides a benchmark equilibrium solution based on the assumption that the private sector forms rational inflation expectations conditional on information available at time \( t \). The RE results are used as a reference point for the discussion about the equilibrium solution obtained under learning in section 4.

3.1 The equilibrium solution

We assume that the central bank does not necessarily benefit from full instrument independence while conducting its optimal policy, implying that the government can intervene in the policy making. Thus the CB needs to take account of a weighted average of its own loss function and the government’s one, depending on the level of
instrument independence. The Lagrangian of the CB is therefore:

\[
L^P = \frac{1}{2} E_t \sum_{i=0}^{+\infty} \beta^i \left[ \left( \mu \phi + \delta^G \right) \pi_{t+i}^2 + \alpha \left( x_{t+i} - \bar{x} \right)^2 \right] \\
- \lambda_{1,t} \left( \pi_t - \beta E_t^* \pi_{t+1} - \kappa x_t - e_t \right) \\
- \lambda_{2,t} \left[ x_t - E_t^* x_{t+1} + \sigma^{-1} (r_t - E_t^* \pi_{t+1}) \right],
\]  

(8)

where \( \lambda_{1,t} \) and \( \lambda_{2,t} \) represent the Lagrangian multipliers respectively associated with the Phillips curve (1) and the IS equation (2).

Differentiating (8) with respect to \( \pi_t, x_t \) and \( r_t \) yields the first-order conditions:

\[
\frac{\partial L^P}{\partial \pi_t} \Rightarrow \lambda_{1,t} = \left( \mu \phi + \delta^G \right) \pi_t, 
\]  

(9)

\[
\frac{\partial L^P}{\partial x_t} \Rightarrow \lambda_{1,t} = - \frac{\alpha}{\kappa} \left( x_t - \bar{x} \right), 
\]  

(10)

\[
\frac{\partial L^P}{\partial r_t} \Rightarrow \lambda_{2,t} = 0.
\]  

(11)

Inserting (9) in (10) gives the trade-off condition for the CB:

\[
x_t = - \frac{\kappa}{\alpha} \left( \mu \phi + \delta^G \right) \pi_t + \bar{x}.
\]  

(12)

The trade-off condition at the RE equilibrium depends on both the negotiation and the inflation aversion of the CB and the government. Furthermore, if there is no negotiation and the CB follows a discretionary monetary policy, then the inflation level is determined by the degree of inflation aversion of the government.

**Proposition 1:** Under discretionary monetary policy, inflation is sensitive to the inflation aversion of the government and the CB and, the degree of negotiation. If the CB is independent, the inflation is only sensitive to the CB’s inflation aversion.
Proof: If \( \mu \to 1 \), then \( x_t = -\frac{\alpha}{\alpha} \delta^G \pi_t + \bar{x} \). If \( \mu = 0 \), \( x_t = -\frac{\alpha}{\alpha} \delta^G \pi_t + \bar{x} \). If \( \mu \to 0 \) and \( \delta^G \to 0 \), then \( x_t = \bar{x} \). These results and equation (1) lead to Proposition 1. \( \square \)

In the extreme case where there is no negotiation and no inflation aversion of the government, i.e., the CB fully depends on the government \( (\mu = 0) \) and the latter prefers stabilizing the output gap \( (\delta^G = 0) \), then the output gap only depends on the overambitious output-gap target.

The system of equations (1) and (12) has one unique RE equilibrium (REE) solution that is non-explosive and called the “minimal state variable” solution (McCallum 1983), in terms of state variable \( e_t \) and \( \bar{x} \). The supposed solution of expected inflation is \( E_\pi = E_\pi + 1 = \zeta_0 + \zeta_1 E_t e_t + 1 = \zeta_0 \). Using the latter and equations (1) and (12) yields the equilibrium solutions of inflation, the output gap and the interest rate rule, under RE:

\[
\pi_t = \frac{\alpha \kappa}{\alpha (1 - \beta) + \kappa^2 (\mu \phi + \delta^G)} \bar{x} + \frac{\alpha}{\alpha + \kappa^2 (\mu \phi + \delta^G)} e_t, \quad (13)
\]

\[
x_t = \frac{\alpha (1 - \beta)}{\alpha (1 - \beta) + \kappa^2 (\mu \phi + \delta^G)} \bar{x} - \frac{\kappa (\mu \phi + \delta^G)}{\alpha + \kappa^2 (\mu \phi + \delta^G)} e_t, \quad (14)
\]

\[
r_t = -\frac{\alpha \sigma (1 - \beta)}{\alpha (1 - \beta) + \kappa^2 (\mu \phi + \delta^G)} \bar{x} + \frac{\sigma \kappa (\mu \phi + \delta^G)}{\alpha + \kappa^2 (\mu \phi + \delta^G)} e_t. \quad (15)
\]

From (13)-(15), notice that the absence of the overambitious output-gap target eliminates the inflation bias and the stabilization bias. Indeed, it yields for \( \bar{x} = 0 \) that:

\[
\pi_t = \frac{\alpha}{\alpha + \kappa^2 (\mu \phi + \delta^G)} e_t, \quad (16)
\]

\[
x_t = -\frac{\kappa (\mu \phi + \delta^G)}{\alpha + \kappa^2 (\mu \phi + \delta^G)} e_t, \quad (17)
\]

\[
r_t = \frac{\kappa \sigma (\mu \phi + \delta^G)}{\alpha + \kappa^2 (\mu \phi + \delta^G)} e_t. \quad (18)
\]

The higher the level of negotiation and the inflation aversion are, the less the cost-
push shock influences the level of inflation.

3.2 The optimal level of instrument independence and conservatism

We can determine the optimal level of instrument independence and conservatism that ensure the absence of stabilization bias and neutralize the effect of the overambitious output-gap target. For that purpose, \( \pi_t \) and \( x_t \) are substituted by their equilibrium values in the government loss function (3):

\[
L^G_t = \frac{1}{2} E_t \sum_{i=0}^{\infty} \beta^i \left\{ \frac{\alpha \kappa}{\alpha (1 - \beta) + \kappa^2 (\mu \phi + \delta^G)} \tilde{x} \right\}^2 + \alpha \left\{ \frac{\alpha (1 - \beta)}{\alpha (1 - \beta) + \kappa^2 (\mu \phi + \delta^G)} \tilde{x} - \tilde{x} \right\}^2.
\]

Differentiating the above loss function with respect to the level of negotiation, \( \mu \), yields \( \mu \) as the function of the level of inflation aversion \( \delta^G \) and conservatism \( \phi \) such as:

\[
\mu = \frac{\beta \delta^G}{\phi (1 - \beta)} \quad (19)
\]

The optimal level of negotiation \( \mu \) for the CB also depends on the inflation aversion of both the CB and the government. Now, looking for the optimal degree of conservatism by differentiating the loss function according to \( \delta^{CB} \), the same condition (19) is obtained. The simulation findings confirm the results of Eijffinger and Hoeberichts (2008), and Hefeker and Zimmer (2011), who show that a high level of instrument independence of the CB does not require a high degree of conservatism in the society but a great dependence of the CB to the government can be counterweighted by the presence of a high inflation aversion of the government.

We can deduce from Figure 1 that the high inflation aversion of the government
Figure 1: Optimal degree of conservatism under RE

associated with the dependence of the CB leads to a very high degree of conservatism.
To examine the direct impact of inflation aversion, we make the restrictive hypothesis
that the government is not inflation-averse and prefers stabilizing the output gap, due
to electoral pressure (Blinder 1996, Goodfriend 2012, and Blinder et al. 2017), and this
leads to Proposition 2.

**Proposition 2:** When the government prefers stabilizing the output-gap to
inflation with $\delta^G = 0$, the optimal negotiation level is zero under RE.

**Proof:** From (19), if $\delta^G \to 0$, then we obtain $\mu = f(\phi) = \frac{3\delta^G}{(1-\beta)} \to 0$. $\square$

When the government has no inflation aversion, the inflation aversion of the central
banker does not matter for setting the optimal negotiation level and thus, the optimal
instrument independence. In this case, the CB is fully dependent on the government.
4 The equilibrium solution under constant-gain learning

This section focuses on the equilibrium solution when agents are learning with a constant-gain algorithm. We study the effects of learning, negotiation and inflation aversion on the optimal monetary policy.

4.1 The Actual Law of Motions

Using the methodology of Molnár and Santoro (2014), we find the ALMs for the inflation, the output gap and the interest rate (Appendix A.4). First, the CB, as under RE, must set its policy interest rate by minimizing the weighted loss function including the government’s and the CB’s respective loss functions, and by including inflation expectations and the output-gap expectations of private agents for current period and
one period ahead. The resulting Lagrangian is:

\[
\min_{\pi_t, x_t, r_t, a_{t+1}, b_{t+1}} L_P = \beta^t E_t \sum_{i=0}^{+\infty} \left\{ \frac{1}{2} \left[ (\mu \phi + \delta^G) \pi_{t+i}^2 + \alpha \mu (x_{t+i} - \bar{x})^2 + (1 - \mu)x_{t+i}^2 \right]
- \lambda_{1,t+i} [\pi_{t+i} - \beta a_{t+i} - \kappa x_{t+i} - e_{t+i}]
- \lambda_{2,t+i} [x_{t+i} - b_{t+i} - \sigma^{-1}(r_{t+i} - a_{t+i})]
- \lambda_{3,t+i} [a_{t+1+i} - a_{t+i} - \gamma_{t+1+i}(\pi_{t+i} - a_{t+i})]
- \lambda_{4,t+i} [b_{t+1+i} - b_{t+i} - \gamma_{t+1+i}(x_{t+i} - b_{t+i})] \right\},
\]

(20)

Differentiating (20) with respect to \(\pi_t, x_t, r_t, a_{t+1}\) and \(b_{t+1}\) yields the following first-order conditions:

\[
\frac{\partial L_P}{\partial \pi_t} = 0 \Rightarrow (\mu \phi + \delta^G) \pi_t - \lambda_{1,t} + \gamma_{t+1}\lambda_{3,t} = 0,
\]

(21)

\[
\frac{\partial L_P}{\partial x_t} = 0 \Rightarrow \alpha (x_t - \bar{x}) + \kappa \lambda_{1,t} - \lambda_{2,t} + \gamma_{t+1}\lambda_{4,t} = 0,
\]

(22)

\[
\frac{\partial L_P}{\partial r_t} = 0 \Rightarrow \sigma^{-1}\lambda_{2,t} = 0,
\]

(23)

\[
\frac{\partial L_P}{\partial a_{t+1}} = 0 \Rightarrow \beta E_t \left[ \lambda_{1,t+1} \beta - \sigma^{-1}\lambda_{2,t+1} + \lambda_{3,t+1}(1 - \gamma_{t+2}) \right] - \lambda_{3,t} = 0,
\]

(24)

\[
\frac{\partial L_P}{\partial b_{t+1}} = 0 \Rightarrow \beta E_t \left[ \lambda_{4,t+1}(1 - \gamma_{t+2}) \right] = \lambda_{4,t}.
\]

(25)

From (25), the only possible value for \(\lambda_{4,t}\) respecting the condition leads to \(\lambda_{4,t} = E_t \lambda_{4,t+1} = 0\). Using the fact that \(\lambda_{4,t} = 0\) and \(\lambda_{2,t} = 0\) from (23), into (22) and into (25) respectively yields the conditions \(\lambda_{1,t} = -\frac{\alpha}{\kappa} x_t + \frac{\alpha}{\kappa} \bar{x}\) and \(\lambda_{3,t} = \beta E_t [-\frac{\alpha}{\kappa} x_{t+1} + \frac{\alpha}{\kappa} \bar{x} + \lambda_{3,t+1}(1 - \gamma_{t+2})]\). We now substitute \(\lambda_{1,t+1}, \lambda_{3,t}\) and \(\lambda_{3,t+1}\) by their expression into (21) and (22) and obtain the intertemporal trade-off condition as:

\[
(\mu \phi + \delta^G) \pi_t + \frac{\alpha}{\kappa} x_t - \frac{\alpha}{\kappa} \bar{x} + \gamma_{t+1}\lambda_{3,t} = 0.
\]

(26)
The term $\gamma_{t+1}\lambda_{3,t}$ is introduced by learning and influences in the trade-off condition for the CB between stabilizing the inflation or the output-gap. Following Molnár and Santoro (2014), we deduce from (6) and (20) that $\gamma_{t+1}$ stands for the marginal effect of an increase in inflation on inflation expectations, $\lambda_{3,t}$ the marginal effect of an increase in inflation expectations on welfare loss at time $t+1$. Learning therefore introduces an intertemporal trade-off for the CB of stabilizing inflation (the output gap) in period $t$ or in period $t+1$. As $\gamma_{t+1} > 0$, the sign of $\lambda_{3,t}$ depends on the sign of inflation expectations, $a_t$, formed in current period $t$. Since the inflation target is set to zero, $a_t$ can either be positive or negative and the sign of $a_t$ depends on the nature of past cost-push shocks. If $a_t$ is positive (negative), $\lambda_{3,t}$ is positive (negative) since an increase in $a_t$ takes inflation expectations further away from (closer to) the inflation target and means lower (higher) social welfare.

Substituting the Lagrangian multiplier $\lambda_{3,t}$ by its value given by (26) into (24), and assuming constant-gain learning yield:

$$\kappa(\mu\phi+\delta^G)\pi_t + \alpha x_t = \beta\kappa(\mu\phi+\delta^G)(1-\gamma)E_t\pi_{t+1} + \alpha\beta[1-\gamma(1-\beta)]E_tx_{t+1} + \alpha\{1-\beta[1-\gamma(1-\beta)]\}\tilde{x}.$$  (27)

Using (1) and (27), the ALM for inflation is obtained as:

$$\pi_t = c^g_{\pi}a_t + \Omega^g_{\pi}\tilde{x} + d^g_{\pi}\tilde{e}_t,$$  (28)

with

$$c^g_{\pi} = \frac{-p_1 - \sqrt{p_1^2 - 4p_2p_0}}{2p_2} > 0,$$  (29)

$$d^g_{\pi} = \frac{\kappa(\mu\phi+\delta^G)+\alpha+\gamma^2(\beta-c_{\pi}^g\phi)(\beta-c_{\pi}^g\phi)^2+\beta(1-\gamma)(\alpha\beta-\alpha\phi+\alpha\phi^2)}{\Phi(y)} > 0,$$  (30)

$$\Omega^g_{\pi} = \frac{\alpha\kappa\{1-\beta[1-\gamma(1-\beta)]\}}{\Phi(y)} > 0.$$  (31)
where \( p_0 = \alpha \beta \{ 1 - \beta (1 - \gamma) [1 - \gamma (1 - \beta)] \} > 0 \),
\[ p_1 = -\kappa^2 (\mu \phi + \delta G) \{ 1 - \beta (1 - \gamma) \} - \alpha (1 - \beta) \{ 1 - \beta [1 - \gamma (1 - \beta)] \} - p_0 - p_2, \]
\[ p_2 = \gamma \beta \{ \kappa^2 (\mu \phi + \delta G) (1 - \gamma) + \alpha [1 - \gamma (1 - \beta)] \} > 0, \]
and
\[ \Phi(y) = \kappa^2 (\mu \phi + \delta G) + \alpha \{ 1 + \beta^2 \gamma [1 - \gamma (1 - \beta)] \} - \beta (\epsilon^g \gamma + 1) \{ (1 - \gamma) [\kappa^2 (\mu \phi + \delta G) + \alpha] + \alpha \gamma \beta \}. \]

Current inflation rises proportionately less than expected inflation since \( 0 < \frac{\alpha \beta}{\alpha + \kappa^2 (\mu \phi + \delta G)} < \frac{\alpha \beta}{\alpha + \kappa^2 (\mu \phi + \delta G)} \) (see Appendix A.3). The CB, when conducting policy, must take into account three dimensions including past shocks embodied in the expected inflation equation (6), the level of negotiation and inflation aversion. The learning effect on the feedback coefficient \( \epsilon^g \) is ambiguous. From (6), an increase in \( \gamma \) yields a positive feedback from current inflation to inflation expectations \( a_{t+1} \), which justifies the CB intervention to lower the feedback coefficient \( \epsilon^g \). Although, an increase in \( \gamma \) can also mitigate the effect of current inflation expectations on future inflation expectations. This, according to (6), means that a greater \( \epsilon^g \) is possible without degrading social welfare.

Using the New Phillips curve (1) into (28), the ALM for the output gap is:
\[ x_t = \epsilon^g x_t a_t + d^g x_t e_t + \Omega^g x_t \tilde{x}, \quad (32) \]

where \( \epsilon^g = -\frac{1}{\kappa} (\beta - \epsilon^g), \; d^g = -\frac{1}{\kappa} (1 - d^g), \) and \( \Omega^g = \frac{1}{\kappa} \Omega^g. \)

Substituting \( x_t \) given by the ALM for the output gap (32) into (2), and using the definition of expected inflation and output gap (6)-(7) yield the ALM for the interest rate:
\[ r_t = \epsilon^g r_t a_t + \delta^g b_t + d^g e_t + \Omega^g r_t \tilde{x}, \quad (33) \]

with \( \epsilon^g = 1 + \frac{\sigma}{\kappa} (\beta - \epsilon^g), \; \delta^g = \sigma, \; d^g = \frac{\sigma}{\kappa} (1 - d^g), \) and \( \Omega^g = -\frac{\sigma}{\kappa} \Omega^g. \) Since \( 0 < \frac{\alpha \beta}{\alpha + \kappa^2 (\mu \phi + \delta G)} < \frac{\alpha \beta}{\alpha + \kappa^2 (\mu \phi + \delta G)} \), the interest-rate rule is active since \( \epsilon^g > 1 \).
4.2 The learning effect on the optimal monetary policy

To show the effect of learning, we compare the feedback coefficients at the learning equilibrium with the corresponding ones at the RE equilibrium. Given that we obtain similar results under constant-gain or decreasing-gain learning since \( \gamma \in (0,1) \) as well as \( \gamma_t \), we focus from now on the results obtained under constant-gain learning. Indeed, as \( \gamma \to 0 \), then \( c^g_\pi = \frac{\alpha \beta}{\alpha + \kappa^2(\mu \phi + \delta G)} \), \( d^g_\pi = \frac{\alpha}{\alpha + \kappa^2(\mu \phi + \delta G)} \), and \( \Omega^g_\pi = \frac{\alpha \kappa}{\alpha + \kappa^2(\mu \phi + \delta G)} \). In this case, the feedback effect of the cost-push shock and the output-gap target are equal under RE (equation (13)), and under learning.

We find also the same feedback coefficients for the ALM for the output gap and the interest rate under both RE and learning. The other possible extreme case with \( \gamma \to 1 \), i.e., when agents are only forming their expectations based on the last values of inflation and the output gap, according to (6)-(7), generates the following feedback coefficients:
\[
    d^g_\pi = \frac{\alpha}{\kappa^2(\mu \phi + \delta G) + \alpha[1 + \beta^2(\beta - c^g_\pi)]}
\]
and
\[
    \Omega^g_\pi = \frac{\alpha \kappa(1 - \beta^2)}{\kappa^2(\mu \phi + \delta G) + \alpha[1 - \beta^2(1 + c^g_\pi - \beta)]}
\]. On the contrary to the case where the learning coefficient allows convergence to the RE equilibrium, i.e., with \( \gamma \to 0 \), when \( \gamma \to 1 \), the feedback coefficients \( d^g_\pi \) and \( \Omega^g_\pi \) ultimately also depend on the feedback coefficient on expected inflation, thus introducing persistence into the model. The extreme case with \( \gamma \to 0 \) is represented in all following simulation figures in red. For standard parameter values, i.e., \( \alpha = 0.048, \kappa = 0.024, \beta = 0.99, \) and \( \sigma = 0.157 \), Figure 3 shows how feedback coefficients evolve according to learning gain for different values of the inflation aversion \( \phi \) and , the negotiation coefficient \( \mu \).
The learning effects are clear from analytical results and simulations. The nature of the deviations of the feedback coefficients from their level under the REE remains unchanged compared to the results found in Molnár and Santoro (2014) and André and Dai (2017).

**Proposition 3:** Learning reduces (increases) deviations from the feedback coefficients on inflation and cost-push shock in the ALM for inflation (the output gap and the interest rate), compared to their level at the RE equilibrium. The deviations of coefficients on the output-gap target decrease with learning in all ALMs.

Proof: From Appendix A.5, we have \( \frac{\partial c^g}{\partial \gamma} < 0, \frac{\partial d^g}{\partial \gamma} < 0, \text{ and } \frac{\partial \Omega^g}{\partial \gamma} < 0. \) Using the definition of \( c_x^g, d_x^g, c_r^g, \text{ and } d_r^g, \Omega_x^g \text{ and } \Omega_r^g, \) it is straightforward to show the sign of their partial derivative with respect to \( \gamma. \) \( \square \)
4.3 The negotiation effect

In the benchmark case, the negotiation is useful for the CB only when the government has an inflation aversion different from zero, no matter if the CB and the government share the same inflation aversion.

One of the main focus in this paper is to consider the impact of negotiation on the optimal monetary policy once private agents are learning. For that matter, both analytical analysis and simulations are driven and they lead to the following proposition.

**Proposition 4:** The deviation of feedback coefficients on inflation and cost-push shock from the level obtained under RE decreases (increases) with the negotiation level in the ALMs for inflation (the output gap and the interest rate) for any level of learning.

*Proof:* From Appendix A.6, we have $\frac{\partial c^g_x}{\partial \mu} < 0$, $\frac{\partial d^g_x}{\partial \mu} < 0$, and $\frac{\partial \Omega^g_x}{\partial \mu} < 0$. Using the definition of $c^g_x$, $d^g_x$, $c^g_r$, $d^g_r$, $\Omega^g_x$, and $\Omega^g_r$, it is straightforward to show the sign of their partial derivative with respect to $\mu$. $\square$

According to Propositions 3 and 4, as $\mu > 0$, the level of relative conservatism interacts the level of instrument-independence. Indeed, the CB’s inflation aversion embodied in $\phi$ in (5) increases the deviation of the feedback coefficients from their level under REE. The closer the negotiation level $\mu$ is close to 1, meaning the CB is fully independent, the higher are the deviations of the feedback coefficients on expected inflation and the cost-push shocks. Furthermore, an increase in learning gain strengthens this trend, making it harder for the CB to manipulate future inflation expectations since from (6)-(7), as $\gamma \to 1$, the expected inflation only depends on past expected inflation.

For the simulations, we adopt the assumption that the government has no interest in stabilizing the inflation and therefore, $\delta^G \to 0$. It allows us to compare with the extreme case under RE.

Since the deviation of the feedback coefficients is greater for the ALM of the output
Figure 4: The feedback coefficients for different level of negotiations and conservatism gap and the interest rate when learning gain, inflation aversion of the government and negotiation level increase, it results that the CB have a more aggressive response in its policy interest rate. More goal-independence means a lower inflation aversion for the government when the CB is highly inflation-averse, or at least, a high enough gap between the inflation aversion of the government and the one of the CB. This is shown in Figure 4 with the ALM for the interest rate, where the feedback coefficients largely exceed 1 as learning, conservatism and the CB independence increase.

**Proposition 5:** For a substantial level of negotiation $\mu$, an increase in learning gain offsets the effect of the overambitious output-gap target on inflation, the output gap and the interest rate. This effect is strengthened by a decrease in the inflation aversion of the government.

Proposition 5 corresponds to the fact that the stabilization bias is thus reduced in presence of a higher learning gain and instrument-independence but a lower inflation aversion of the government. Thus, optimal monetary policy corresponds to a higher level of instrument-independence and a higher level of relative conservatism, i.e., the government and the CB should not share the same high inflation aversion level. Propo-
Figure 5: Feedback coefficients when $\delta^G = \delta^{CB} \to 1$

sition 5 is a distinct feature of learning since according to Lippi (2000), under RE, the stabilization bias is increased when the instrument-independence and the relative conservatism is high.

Figure 5 represents the feedback coefficients for each ALM when the CB and the government share the same inflation aversion that is set at a very high level.

Another interesting result is that negotiation does not influence the level of feedback coefficients when the inflation aversion is high for both institutions, which corroborates the result obtained in Proposition 2 under RE. Thus, from those results we can deduce the following proposition.

**Proposition 6:** When the relative conservatism is null for $\delta^G = \delta^{CB} \to 1$, the level of negotiation has no effect on the optimal monetary policy. Only an increase in learning gain reduces (increases) the deviation of the feedback coefficients on expected inflation and cost-push shocks in the ALM for inflation (the output gap and the interest rate) from their level under RE.
When inflation aversion is high for the CB and the government and the degree of relative conservatism is low, the sensitivity of feedback coefficients is higher than when the CB and the government share a low inflation aversion (Figure 4).

5 Decreasing-gain learning

We here relax the assumption of constant-gain learning to introduce decreasing-gain learning into (6) and (7), i.e., $\gamma_t = \frac{1}{t}$. Assuming that private agents are forming expectations through decreasing-gain learning amounts to say that agents are giving increasingly more weight to present information than to past one over time; this assumption is related to the idea that private agents are experiencing a constrained memory. Decreasing gain yields the following intertemporal condition and ALMS for inflation, the output gap and the interest rate (see Appendix B.1):

$$\kappa \frac{(\mu \phi + \delta G)}{\gamma_{t+1}} \pi_t + \frac{\alpha}{\gamma_{t+1}} x_t + \alpha \left[ \beta \frac{(1-\gamma_{t+2})}{\gamma_{t+2}} + \beta^2 - \frac{1}{\gamma_{t+1}} \right] \tilde{x} = \beta \kappa \left( \mu \phi + \delta G \right) \frac{(1-\gamma_{t+2})}{\gamma_{t+2}} E_t \pi_{t+1} + \alpha \beta \left[ \frac{(1-\gamma_{t+2})}{\gamma_{t+2}} + \beta \right] E_t x_{t+1} \quad (34)$$

$$\pi_t = c_{\pi,t}^{dg} a_t + c_{\pi,t}^{da} e_t + \Omega_{\pi,t}^{dg} \tilde{x}, \quad (35)$$

with

$$c_{\pi,t}^{dg} = \frac{\beta \left\{ \alpha - (1-\gamma_{t+1}) \left[ \alpha (1+\gamma_{t+1}) \beta (\beta - c_{\pi,t+1}^{dg}) - c_{\pi,t+1}^{dg} \kappa^2 (\mu \phi + \delta G) \right] \right\}}{\alpha + \kappa^2 (\mu \phi + \delta G) \left( 1-\beta \gamma_{t+1} c_{\pi,t+1}^{dg} \right) + \alpha \beta \gamma_{t+1} (1+\beta \gamma_{t+1}) \left( \beta - c_{\pi,t+1}^{dg} \right)},$$

$$d_{\pi,t}^{dg} = -\frac{\alpha \kappa \left[ \left( 1-\beta \gamma_{t+1} c_{\pi,t+1}^{dg} \right) + \alpha \beta \gamma_{t+1} (1+\beta \gamma_{t+1}) \left( \beta - c_{\pi,t+1}^{dg} \right) \right]}{\alpha + \kappa^2 (\mu \phi + \delta G) \left( 1-\beta \gamma_{t+1} c_{\pi,t+1}^{dg} \right) + \alpha \beta \gamma_{t+1} (1+\beta \gamma_{t+1}) \left( \beta - c_{\pi,t+1}^{dg} \right)},$$

and

$$\Omega_{\pi,t}^{dg} = -\frac{-\alpha \kappa \left[ (1-\beta) \gamma_{t+1} \beta^2 + \Omega_{\pi,t+1}^{dg} \beta \left( \kappa^2 (\mu \phi + \delta G) + \alpha (1+\beta \gamma_{t+1}) \right) \right]}{\alpha + \kappa^2 (\mu \phi + \delta G) \left( 1-\beta \gamma_{t+1} c_{\pi,t+1}^{dg} \right) + \alpha \beta \gamma_{t+1} (1+\beta \gamma_{t+1}) \left( \beta - c_{\pi,t+1}^{dg} \right)}.$$

Similarly, substituting $\pi_t$ given by (35) into (1) leads to the ALM for the output
gap:

\[ x_t = c_{x,t}^d a_t + d_{x,t}^d e_t + \Omega_{x,t}^d \tilde{x}, \]  

where \( c_{x,t}^d = -\frac{1}{\kappa} (\beta - \alpha \pi_t) \), \( d_{x,t}^d = -\frac{1}{\kappa} (1 - \alpha \pi_t) \), and \( \Omega_{x,t}^d = \frac{1}{\kappa} \Omega_{\pi,t}^d \).

Substituting \( x_t \) given by (36) into (2) and taking account of the definition of expected inflation and output gap (6)-(7) with decreasing gain yield the ALM for the interest rate:

\[ r_t = c_{r,t}^d a_t + \delta_{r,t}^d b_t + d_{r,t}^d e_t + \Omega_{r,t}^d \tilde{x}, \]  

with \( c_{r,t}^d = 1 + \frac{\sigma}{\kappa} (\beta - \alpha \pi_t) \), \( \delta_{r,t}^d = \sigma \), \( d_{r,t}^d = \frac{\sigma}{\kappa} (1 - \alpha \pi_t) \), and \( \Omega_{r,t}^d = -\frac{\sigma}{\kappa} \Omega_{\pi,t}^d \). Given that \( 0 < c_{\pi,t}^d < \frac{\alpha \beta}{a + \kappa^2 (\mu \phi + \beta^2 \gamma)} \), we have \( (\beta - c_{\pi,t}^d) > 0 \). As under constant-gain learning, the interest-rate rule is active since \( c_{r,t}^d > 1 \).

The results obtained under decreasing-gain learning as times goes by remain equivalent to those obtained under constant-gain learning since from Appendix B.2., for \( t > 1 \), \( c_{\pi,t}^d \) exhibits the same characteristics as \( c_{\pi}^g \), obtained for a constant gain, \( \gamma \). We can thus generalize that Propositions 3 to 7 are also valid when private agents are forming expectations through the decreasing-gain learning process.

6 The optimal level of independence and conservatism

Under RE, the level of independence is positively correlated with conservatism, as it is clear from (19). This relationship will be substantially affected by learning that leads the CB to be more aggressive.

The government signs a long-term contract with the central bank to improve the social welfare without accounting for the short-run issue of monetary policy, including the manipulation of private expectations to improve the short-run macroeconomic sta-
bilization. It takes \( E_{t-1} \pi_t = E_t \pi_{t+1} = E_t a_t \), then inserting the latter into (28) gives
\[ E_t \pi_{t+1} = c^{cg}_\pi E a_t + \Omega^{cg}_\pi \tilde{x}, \]
that yields the expected inflation on the long-term approach:
\[ E_t \pi_{t+1} = \frac{\Omega^{cg}_\pi}{1 - c^{cg}_\pi} \tilde{x}. \quad (38) \]

Similarly, using the definition \( c^g_x = -\frac{\beta - c^{cg}_\pi}{\kappa} \), \( \Theta^{cg}_x = \frac{1}{\kappa} \Theta^{cg}_\pi \) and \( \Omega^{cg}_x = \frac{1}{\kappa} \Omega^{cg}_\pi \) into (32) leads to:
\[ E_t x_{t+1} = \frac{\Omega^{cg}_\pi (1 - \beta)}{\kappa (1 - c^{cg}_\pi)} \tilde{x}. \quad (39) \]

Substituting \( E \pi_t \) and \( E x_t \) given respectively by (38) and (39) into the government loss function (3) gives:
\[ L^s_t = L^G_t = \delta^G \left( \frac{\Omega^{cg}_\pi}{1 - c^{cg}_\pi} \tilde{x} \right)^2 + \left( \frac{\Omega^{cg}_\pi (1 - \beta)}{\kappa (1 - c^{cg}_\pi)} \tilde{x} \right)^2. \quad (40) \]

Given the complex effects of \( \mu \) and \( \phi \) on the social welfare, it is impossible to solve analytically the optimal combination of \( \mu \) and \( \phi \) by minimizing (40). To guess their relationship by examining the effect of \( \delta^G \) on the social welfare obtained from the derivatives in A.7 and from the simulations in Figure 6 for different values of \( \mu \), i.e., \( \mu = 0 \) and \( \mu = 1 \).
Figure 6: Social loss with different negotiation levels ($\mu = 0$ and $\mu = 1$)

Under learning, if the society is conservative, i.e., when the government is not inflation averse and the CB is highly inflation-averse, i.e., $\delta^G = 0$, the welfare is low, while, if the society is not conservative in the case where the government and the CB share a high inflation aversion ($\delta^G = \delta^{CB} = 1$), the welfare loss considerably increases. Compared to the RE equilibrium, the learning increases the welfare loss for any given level of negotiation and inflation aversion and it is noteworthy that the welfare loss is greater when the CB is fully instrument-independent under learning.

**Proposition 7:** An increase in instrument-independence and learning contribute to raise the welfare loss, for any level of inflation aversion of the government. Furthermore, when the CB fully depends on the government ($\mu = 0$), the welfare loss is null if the inflation aversion for the government is null.
The welfare loss is greater under learning when the CB is fully instrument-independent. The coefficients increase with learning gain, and independence in both ALMs for inflation and the output gap, the CB should be even more aggressive. The absence of any measures, or institutional design, to correct the increasing welfare loss explains why the CB should have a more aggressive response of its interest rate policy as the CB becomes more independent from the government, as the learning gain increases and the CB’s inflation aversion is high. Therefore, the optimal level of negotiation according to simulations is when the CB fully depends on the government ($\mu = 0$) and when the government has no interest in stabilizing inflation ($\delta^G = 0$). Implementing an inflation contract (Walsh 1995, 2003) between the government and the CB could be an interesting alternative if the CB has to remain hawkish for high levels of learning.

7 Level of negotiation and inflation contract

Instead of appointing a conservative central banker to deal with the time-inconsistency problem, the government can impose an inflation contract that could discipline the central banker in his/her conduct of policy when agents are learning. Following section 4, we introduce the assumption that the CB has to deal with a linear inflation contract, set by the government, to examine if instrument independence can be compatible with learning and can offset the inflation bias and the stabilization bias (Persson and Tabellini 1993, Walsh 1995, Dai and Spyromitros 2012). This “contracting” approach is based on the idea that the government, as the principal, designs an optimal incentive

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6Inflation contracts compared to other institutional arrangements are more efficient to eliminate those two biases, combined with CB independence under RE (Frattiani et al. 1997). For a more detailed analysis, Candel-Sánchez and Campoy-Miñarro (2004) and Chortareas and Miller (2007) have discussed the cost of inflation contract and its consequences. Several studies have further examined how linear inflation contracts are impacted by the central banker’s responsiveness to incentive schemes (Chortareas and Miller 2003), by uncertainty about the CB’s output target (Muscatelli, 1999), and by preference weights on the objectives of inflation and output stabilization set by the CB (Beetsma and Jensen 1998, and Muscatelli 1998, 1999).
scheme for the CB such that monetary policy outcomes are equivalent to those under credible commitment. Generally, such scheme includes an efficient transfer mechanism, also called inflation penalty, that deters the central banker’s tendency to conduct a more accommodative monetary policy by sufficiently increasing the welfare costs of inflation surprise.

7.1 The inflation contract

The government, as benevolent social planner, imposes on the central banker an inflation contract stipulating that she or he receives a monetary transfer payment from the government according to a rule (Walsh 1995), i.e.,

$$T = \tau_0 - \tau \pi_t.$$  \hfill (41)

To conduct optimal discretionary policy, the CB solves the following problem:

$$L^P = \mu L^G_t + (1 - \mu) L^C_t$$
$$= \frac{1}{2} E_t \sum_{i=0}^{\infty} \beta^i \left[ (\mu \phi + \delta^G) \pi_{t+i}^2 - \mu \xi (\tau_0 - \tau \pi_{t+i}) + \alpha (x_{t+i} - \bar{x})^2 \right].$$  \hfill (42)

The parameter $\xi$ gives the level of incentive that makes the central banker care about monetary transfer payment, $\tau_0$ is fixed such that the CB’s participation is ensured and $\tau$ is the penalty rate associated with the level of inflation (Walsh 1995). Such a payment is either a synonym of the direct income of the central banker or the budget of the CB.
7.2 Benchmark equilibrium solution and optimal inflation contract

Minimizing the loss function with respect to inflation, the output gap and the interest rate, we find the targeting rule for the CB:

\[ x_t = -\frac{\kappa}{\alpha} (\mu \phi + \delta^G) \pi_t - \frac{\kappa \mu}{\alpha} \frac{1}{2} \xi \tau + \bar{x}. \] \hspace{1cm} (43)

Using the undetermined coefficients method solved from (1) and (43), we find the RE equilibrium solution where:

\[ \pi_t = \frac{\alpha \kappa}{\alpha (1-\beta) + \kappa^2 (\mu \phi + \delta^G)} \bar{x} - \frac{\kappa^2 \mu}{\alpha (1-\beta) + \kappa^2 (\mu \phi + \delta^G)} \frac{\xi}{\alpha + \kappa^2 (\mu \phi + \delta^G)} \tau + \frac{\alpha}{\alpha + \kappa^2 (\mu \phi + \delta^G)} \xi \tau; \] \hspace{2cm} (44)

\[ x_t = \frac{\alpha (1-\beta)}{\alpha (1-\beta) + \kappa^2 (\mu \phi + \delta^G)} \bar{x} - \frac{\kappa \mu (1-\beta)}{\alpha (1-\beta) + \kappa^2 (\mu \phi + \delta^G)} \frac{\xi}{\alpha + \kappa^2 (\mu \phi + \delta^G)} \tau - \frac{\kappa (\mu \phi + \delta^G)}{\alpha + \kappa^2 (\mu \phi + \delta^G)} \xi \tau + \frac{\kappa (\mu \phi + \delta^G)}{\alpha + \kappa^2 (\mu \phi + \delta^G)} \xi \tau; \] \hspace{2cm} (45)

Based on the RE equilibrium, we solve the government’s minimization problem for the optimal inflation penalty rate:

\[ \tau = 2 \alpha \kappa \left( \frac{\mu \phi + \delta^G}{\alpha (1-\beta)^2 + \kappa^2 \delta^G} \right) \left( 1 - \beta \right) - \delta^G \frac{1}{\mu \xi} \bar{x}. \] \hspace{2cm} (46)

Notice that when the output-gap target is null, then the optimal inflation penalty rate is also equal to zero. The absence of the output-gap target leads to the absence of both stabilization bias and inflation bias, explaining why the inflation contract is only needed when there exists an overambitious output-gap target. Figure 7 shows that the optimal inflation penalty rate is higher for a higher degree of negotiation and conservatism.
Proposition 9: The inflation penalty is positively impacted by the negotiation level and tends to zero as $\mu \to 1$.

Proof: Differentiating $\tau$ in (46) with respect to $\mu$ yields:

$$\frac{\partial \tau}{\partial \mu} = 2 \frac{\alpha \beta \delta G \kappa [\alpha (1-\beta)^2 + \kappa^2 \delta G]}{\xi \{ [\alpha (1-\beta)^2 + \kappa^2 \delta G] \mu \}^2} \tilde{x} > 0. \blacksquare$$

Electoral considerations are directly explaining why under RE, the linear contract the government sets leads the central banker to receive a positive monetary payment $T$ from (41) since $\tau < 0$, when the CB depends on the government ($\mu = 0$) and if the society is not conservative, i.e., does not care much about the level of inflation ($\delta^G = 0$).
7.3 The ALMs for inflation, the output-gap and the interest rate

Similarly to subsection 4.1, we obtain by minimizing (42) with respect to $\pi_t$, $x_t$, $r_t$, $a_{t+1}$ and $b_{t+1}$ the ALMs for inflation, the output gap and the interest rate:

$$\pi_t = c^g_{\pi} a_t + d^g_{\pi} e_t + \Omega^g_{\pi} \bar{x} + \Gamma^g_{\pi} \xi_t,$$  \hspace{1cm} (47)

with

$$c^g_{\pi} = \frac{-p_1 - \sqrt{p_1^2 - 4p_2p_0}}{2p_2} > 0,$$  \hspace{1cm} (48)

$$d^g_{\pi} = \frac{\kappa^2(\mu \phi + \delta^G) + \alpha + \gamma^2 \beta^2 (\beta - c^g_{\pi}) + \beta \gamma (1 - \gamma)(\alpha \beta - \alpha + \kappa^2(\mu \phi + \delta^G) c^g_{\pi})}{\Phi} > 0,$$  \hspace{1cm} (49)

$$\Omega^g_{\pi} = \frac{\alpha \kappa \{1 - \beta \{1 - \gamma (1 - \beta)\}\}}{\Phi} > 0,$$  \hspace{1cm} (50)

$$\Gamma^g_{\pi} = -\frac{1}{2} \frac{\kappa^2 \gamma \mu [1 - \beta (1 - \gamma)]}{\Theta} < 0.$$  \hspace{1cm} (51)

where $p_0 = \alpha \beta \{1 - \beta (1 - \gamma) [1 - \gamma (1 - \beta)]\} > 0$, $p_1 = -\kappa^2(\mu \phi + \delta^G)[1 - \beta (1 - \gamma)] - \alpha(1 - \beta)[1 - \beta (1 - \gamma) [1 - \gamma (1 - \beta)]] - p_0 - p_2$, $p_2 = \gamma \beta \{\kappa^2(\mu \phi + \delta^G)(1 - \gamma) + \alpha[1 - \gamma (1 - \beta)]\} > 0$, $\Phi = \kappa^2(\mu \phi + \delta^G) + \alpha \{1 + \beta^2 \gamma [1 - \gamma (1 - \beta)]\} - \beta (c^g_{\pi})^2 + 1\} (1 - \gamma)[\kappa^2(\mu \phi + \delta^G) + \alpha] + \alpha \gamma \beta \}$$,

$\Theta = \kappa^2(\mu \phi + \delta^G)[1 - \beta (1 - \gamma) - c^g_{\pi} \gamma \beta (1 - \gamma)] + \alpha \{1 - \beta [1 - \gamma (1 - \beta)][1 - \beta \gamma (1 - c^g_{\pi})]\}$.

It is noteworthy that feedback coefficients $c^g_{\pi}$, $d^g_{\pi}$ and $\Omega^g_{\pi}$ are identical to the case where the monetary policy was not subject to a linear inflation contract. The novelty introduced in the ALMs for inflation, the output gap and the interest rate is that the penalty negatively influences the level of inflation and the output gap. Furthermore $\Gamma^g_{\pi}$, the feedback coefficient on the penalty, can be set to zero when agents are no longer learning, i.e., when $\gamma = 0$ but it will depend on the negotiation level, when $\gamma = 1$ that is when agents are forecasting inflation only taking account of past inflation since

$$\Gamma^g_{\pi} = -\frac{1}{2} \frac{\kappa^2 \mu}{\kappa^2(\mu \phi + \delta^G) + \alpha[1 - \beta^2 (1 - \beta + c^g_{\pi})]}.$$
Using (47) into (1), we obtain the ALM for the output gap:

\[ x_t = c^g_x a_t + d^g_x e_t + \Omega^g_x \bar{x} + \Gamma^g_x \xi_t, \quad (52) \]

where \( c^g_x = -\frac{1}{\kappa} (\beta - c^g_\pi) \), \( d^g_x = -\frac{1}{\kappa} (1 - d^g_\pi) \), \( \Omega^g_x = \frac{1}{\kappa} \Omega^g_\pi \), and \( \Gamma^g_x = \frac{\Gamma^g_\pi}{\kappa} \).

Using the New Keynesian IS equation (2) into (52) yields the ALM for the interest rate:

\[ r_t = c^g_r a_t + \delta^g_r b_t + d^g_r e_t + \Omega^g_r \bar{x} + \Gamma^g_r \xi_t, \quad (53) \]

with \( c^g_r = 1 + \sigma \frac{\beta - c^g_\pi}{\kappa} \), \( \delta^g_r = \sigma \), \( d^g_r = \sigma \frac{1 - d^g_\pi}{\kappa} \), \( \Omega^g_r = -\frac{\sigma}{\kappa} \Omega^g_\pi \) and \( \Gamma^g_r = -\frac{\sigma}{\kappa} \Gamma^g_\pi \).

Figure 7 shows how the feedback coefficients evolve with the level of negotiation and the relative conservatism in two extreme cases. First, the CB and the government share the same inflation aversion, which yields a relative conservatism that is null when \( \phi = 0 \) and, secondly, the relative conservatism is set to maximum with \( \phi = 1 \), when the CB and the government have opposite inflation aversion where \( \delta^{CB} = 1 \) and \( \delta^G = 0 \).

Figure 8: Feedback coefficients with different negotiation levels (\( \phi = 0 \) and \( \phi = 1 \))
7.4 The optimal inflation contract under learning

The government can either sign a long-term contract with the central banker, that is to be embedded in the CB’s statutes, or alternatively a short-term contract with the central banker, whose major interest resides in being periodically reviewed and adjusted in an economic environment where private agents are learning. These two approaches differ in that the first one does not take account of the evolution of private expectations formed with learning while the second does. Moreover, the long-term contract entails a sole contract for every forthcoming central banker, whereas the short-term contract can be regarded as an instrument allowing the government to oversee the central banker in each period. Closed-form solutions are given for both type of contracts but only the long-term contract ensures the convergence of the penalty rate towards the RE equilibrium. Therefore we examine more in detail only the long-term one.

Long-term central bank contracting

The government signs a long-term contract with the central bank to improve the steady-state social welfare without taking account of the short-run issue of monetary policy, which includes the manipulation of private expectations to improve the short-run macroeconomic stabilization. In this case, the central banker is free to manage the intertemporal trade-off introduced by learning. The government assumes $E_t \pi_{t+1} = E_t a_t$. Furthermore, the steady state is defined as $\pi_{t+i} = \pi_t = a_{t+i} = a_t$ and $e_{t+i} = e_{t+i} = 0$, then the social loss function can be rewritten using these steady-state conditions, the expectation of the ALM for inflation (47), and the fact that $\epsilon_x = -\frac{\beta - \epsilon_x}{\beta - \gamma}, \Gamma_x = \frac{1}{\kappa} \Gamma_x$.
and $Q_{x}^{cg} = \frac{1}{\kappa^{2}}Q_{\pi}^{cg}$ from (52), it gives (C.5):

$$L_{t}^{s} = \frac{1}{2}E_{t}\sum_{i=0}^{+\infty} \beta^{i} \left[ \delta^{G}\pi_{t+i}^{2} + \alpha (x_{t+i} - \bar{x})^{2} \right]$$

$$= \frac{1}{2(1 - \beta)} \left[ \delta^{G} \left( \frac{\Gamma_{\pi}^{cg} \xi}{1 - c_{\pi}^{cg}} \tau + \frac{Q_{\pi}^{cg} \Omega_{\pi}^{cg}}{1 - c_{\pi}^{cg}} \bar{x} \right)^{2} + \alpha \left( \frac{(1 - \beta) \xi \Gamma_{\pi}^{cg} \Omega_{\pi}^{cg}}{\kappa(1 - c_{\pi}^{cg})} \tau + \frac{(1 - \beta) \Omega_{\pi}^{cg}}{\kappa(1 - c_{\pi}^{cg})} \bar{x} - \bar{x} \right)^{2} \right]$$

Differentiating this loss function with respect to $\tau$ yields a first-order condition that is solved to obtain:

$$\tau = \frac{\alpha \kappa (1 - \beta)(1 - c_{\pi}^{cg}) - \left[ \alpha(1 - \beta)^{2} + \kappa^{2}\delta^{G} \right] \Omega_{\pi}^{cg} \xi}{\left[ \alpha(1 - \beta)^{2} + \kappa^{2}\delta^{G} \right] \Gamma_{\pi}^{cg} \xi} \bar{x}.$$  \hspace{1cm} (54)

We demonstrate that the optimal inflation penalty rate at the equilibrium when $\gamma = 0$ is exactly the same as in the RE equilibrium (see equation (46)).

$$\tau = 2\alpha \kappa \left( \mu \phi + \delta^{G} \right)(1 - \beta) - \delta^{G} \bar{x}.$$  \hspace{1cm} (55)

Figure 8 illustrates the evolution of the optimal inflation penalty rate depending on the degree of conservatism. From simulations, the higher the conservatism is, the faster the optimal inflation penalty rate converges to zero as learning increases for a high enough level of instrument independence. The degree of CB conservatism slightly affects the level of optimal inflation penalty rate under the long-term contract. The case of full independence is not examined in this paper on the optimal inflation penalty rate since it has already been studied in André and Dai (2017) where we found that the optimal inflation penalty rate is always positive and decreasing towards zero as learning gain increases.
Proposition 10: An increase in learning and negotiation both decreases the level of the inflation penalty rate to zero.

The optimal inflation penalty rate is even more positively sensitive to negotiation level when learning gain is high. On the contrary to the full instrument-independence case mentioned in the above, the optimal inflation penalty rate is negative when the CB is instrument- and goal-dependent whereas it tends to positive values as the instrument independence increases. The optimal inflation penalty rate exhibits a number of positive values that are greater under learning than under RE, when $\phi = 1$. This phenomenon is strengthened by a lower learning gain. Furthermore, if the CB is more goal-dependent ($\phi \to 0$), then the optimal inflation penalty rate is always positive for a given level of instrument-independence.

Proposition 11: If simultaneously, the society shows a high degree of relative conservatism $\phi$ and the CB is instrument-dependent, then the opti-
mal inflation penalty rate increasingly hits negative values. This effect is strengthened by a decrease in learning gain.

For a given level of negotiation and learning, the level of inflation penalty rate decreases with conservatism. But there is a countereffect where more instrument-independence, i.e., more negotiation, leads the inflation penalty rate to tend to zero. Therefore, conservatism and negotiation have a cumulated effect on the inflation penalty rate that is ambiguous under learning. The higher the learning, the less ambiguous is this effect. The effect of negotiation dominates the effect of conservatism.

8 Conclusion

The recent financial and economic crisis has revived debates on the desirable conduct of monetary policy, including the issues of instrument- and goal-independence. This paper shows that the central banker should be hawkish as learning, instrument- and goal-independence increase. Furthermore, learning reinforces the results obtained by Hugues Hallett and Weymark (2005) and, Eijffinger and Hoeberichts (2008) since more instrument-independence does not impose a higher degree of relative conservatism for the society, when an inflation contract is imposed on the central banker. The response of inflation, the output gap and the nominal interest rate to cost-push shocks and to a change in inflation expectations under adaptive learning is amplified by an increase of independence and a decrease of inflation conservatism. We have considered in this paper that instrument-independence and relative, conservatism and learning introduce a new trade-off for the central bank between offsetting inflation and stabilization biases and decreasing welfare loss while learning gain reinforces the instrument-independence effect. Introducing different instrument-independence levels leads to an optimal inflation penalty rate that can either be positive, null or negative according
to learning gain and goal-independence level and, tempers the results obtained in the full instrument independence case (André and Dai 2017) where the inflation penalty rate is positive. Furthermore, it corroborates the fact that when the CB is goal- and instrument-dependent, it has to cope with the government facing electoral pressures, explaining why the optimal inflation penalty rate is negative.
A Appendix for constant-gain learning

A.1 Minimization of the Lagrangian and FOCs

The CB sets its policy interest rate by minimizing the weighted loss function including the government’s and the CB’s respective loss functions, taking account of inflation and output-gap expectations of private agents. The resulting Lagrangian is:

\[
\min_{\pi_t, x_t, r_t, a_{t+1}, b_{t+1}} L^P = \beta E_t \sum_{i=0}^{+\infty} \left\{ \frac{1}{2} \left[ (\mu \phi + \delta^G) \pi_{t+i}^2 + \alpha \mu (x_{t+i} - \tilde{x})^2 + (1 - \mu) x_{t+i}^2 \right] - \lambda_{1,t+i} [\pi_{t+i} - \beta a_{t+i} - \kappa x_{t+i} - e_{t+i}] 
- \lambda_{2,t+i} [x_{t+i} - b_{t+i} - \sigma^{-1} (r_{t+i} - a_{t+i})], 
- \lambda_{3,t+i} [a_{t+1+i} - a_{t+i} - \gamma_{t+1+i} (\pi_{t+i} - a_{t+i})] 
- \lambda_{4,t+i} [b_{t+1+i} - b_{t+i} - \gamma_{t+1+i} (x_{t+i} - b_{t+i})] \right\},
\] (A.1)

Differentiating (A.1) with respect to \( \pi_t, x_t, r_t, a_{t+1} \) and \( b_{t+1} \) yields the following first-order conditions:

\[
\frac{\partial L^P}{\partial \pi_t} = 0 \Rightarrow (\mu \phi + \delta^G) \pi_t - \lambda_{1,t} + \gamma_{t+1} \lambda_{3,t} = 0 \quad (A.2)
\]

\[
\frac{\partial L^P}{\partial x_t} = 0 \Rightarrow \alpha (x_{t+i} - \tilde{x}) + \kappa \lambda_{1,t} - \lambda_{2,t} + \gamma_{t+1} \lambda_{4,t} = 0 \quad (A.3)
\]

\[
\frac{\partial L^P}{\partial r_t} = 0 \Rightarrow \sigma^{-1} \lambda_{2,t} = 0 \quad (A.4)
\]

\[
\frac{\partial L^P}{\partial a_{t+1}} = 0 \Rightarrow \beta E_t [\lambda_{1,t+1} \beta - \sigma^{-1} \lambda_{2,t+1} + \lambda_{3,t+1} (1 - \gamma_{t+2})] - \lambda_{3,t} = 0 \quad (A.5)
\]

\[
\frac{\partial L^P}{\partial b_{t+1}} = 0 \Rightarrow \beta E_t [\lambda_{4,t+1} (1 - \gamma_{t+2})] = \lambda_{4,t} \quad (A.6)
\]

From (25), the only possible value for \( \lambda_{4,t} \) respecting the condition leads to \( \lambda_{4,t} = E_t \lambda_{4,t+1} = 0 \). Using the fact that \( \lambda_{1,t} = 0 \) and \( \lambda_{2,t} = 0 \) from (23), into (22) and into (25) respectively yields the conditions \( \lambda_{1,t} = -\frac{\alpha}{\kappa} x_t + \frac{\alpha}{\kappa} \tilde{x} \) and \( \lambda_{3,t} = \beta E_t [-\frac{\alpha^{2}}{\kappa} x_{t+1} + \frac{\alpha \beta}{\kappa} \tilde{x} + \)}
\[ \lambda_{3,t+1}(1 - \gamma_{t+2}) \]. We substitute \( \lambda_{1,t+1}, \lambda_{3,t} \) and \( \lambda_{3,t+1} \) by their expression into (A.5) to obtain the intertemporal condition:

\[
(\mu \phi + \delta^G) \pi_t + \frac{\alpha}{\kappa} x_t - \frac{\alpha}{\kappa} \tilde{x} + \gamma_{t+1} \lambda_{3,t} = 0, \tag{A.7}
\]

where \( \gamma_{t+1} \) can either be constant or decreasing with time.

Replacing \( \lambda_{3,t} \) and \( \lambda_{3,t+1} \) by their values yields:

\[
\kappa \left( \frac{(\mu \phi + \delta^G) \pi_t + \alpha}{\gamma_{t+1}} x_t \right) = \alpha \left( \beta \left( \frac{1 - \gamma_{t+2}}{\gamma_{t+2}} \right) + \beta^2 - \frac{1}{\gamma_{t+1}} \right) \tilde{x} + \beta \kappa \left( \mu \phi + \delta^G \right) \left( \frac{1 - \gamma_{t+2}}{\gamma_{t+2}} \right) E_t \pi_{t+1} + \alpha \left( \beta \left( \frac{1 - \gamma_{t+2}}{\gamma_{t+2}} \right) + \beta^2 \right) E_t x_{t+1}. \tag{A.8}
\]

Considering constant learning gain, i.e., \( \gamma_{t+1} = \gamma_t = \gamma \) and we obtain the intertemporal condition:

\[
\kappa (\mu \phi + \delta^G) \pi_t + \alpha x_t = \beta \kappa \left( \mu \phi + \delta^G \right) (1 - \gamma) E_t \pi_{t+1} + \alpha \left( 1 - \gamma (1 - \beta) \right) E_t x_{t+1} + \alpha \left( 1 - \beta [1 - \gamma (1 - \beta)] \right) \tilde{x} \tag{A.9}
\]

**A.2 The stability of the system**

Rewriting the Phillips curve (1) while substituting the expected output gap in the next period \( E_t x_{t+1} \) by \( a_t \) as defined in (6) yields the following two equations:

\[
x_t = \frac{1}{\kappa} \bar{\pi}_t - \frac{\beta}{\kappa} a_t - \frac{1}{\kappa} e_t, \tag{A.10}
\]

and

\[
E_t x_{t+1} = \frac{1}{\kappa} E_t \bar{\pi}_{t+1} - \frac{\beta}{\kappa} [a_t + \gamma_{t+1}(\pi_t - a_t)]. \tag{A.11}
\]

Substituting \( x_t \) and \( x_{t+1} \) respectively given by (A.10) and (A.11) into (A.9) and
arranging the terms lead to:

\[
\beta \kappa (\mu \phi + \delta^G) (1 - \gamma) E_t \pi_{t+1} = \kappa (\mu \phi + \delta^G) \pi_t + \alpha \left[ \frac{1}{\kappa} \pi_t - \frac{\beta}{\kappa} a_t - \frac{1}{\kappa} e_t \right] \\
- \alpha \beta [1 - \gamma (1 - \beta)] E_t \left[ \frac{1}{\kappa} \pi_{t+1} - \frac{\beta}{\kappa} (a_{t+1}) \right] \\
- \alpha \{1 - \beta [1 - \gamma (1 - \beta)]\} \bar{x}
\]

We then transform the above equation to obtain this final expression of expected inflation:

\[
E_t \pi_{t+1} = A_{11} \pi_t + A_{12} a_t + A_{13} \bar{x} + P_1 e_t, \quad (A.12)
\]

where

\[
A_{11} \equiv \frac{\kappa^2 (\mu \phi + \delta^G) + \alpha \{1 + \beta^2 \gamma [1 - \gamma (1 - \beta)]\}}{\beta \{\kappa^2 (\mu \phi + \delta^G) (1 - \gamma) + \alpha [1 - \gamma (1 - \beta)]\}^2}, \quad (A.13)
\]

\[
A_{12} \equiv - \frac{\alpha \beta \{1 - \beta (1 - \gamma) [1 - \gamma (1 - \beta)]\}}{\beta \{\kappa^2 (\mu \phi + \delta^G) (1 - \gamma) + \alpha [1 - \gamma (1 - \beta)]\}^2}, \quad (A.14)
\]

\[
A_{13} \equiv - \frac{\alpha \kappa \{1 - \beta [1 - \gamma (1 - \beta)]\}}{\beta \{\kappa^2 (\mu \phi + \delta^G) (1 - \gamma) + \alpha [1 - \gamma (1 - \beta)]\}^2}, \quad (A.15)
\]

\[
P_1 \equiv - \frac{\alpha}{\beta \{\kappa^2 (\mu \phi + \delta^G) (1 - \gamma) + \alpha [1 - \gamma (1 - \beta)]\}^2}. \quad (A.16)
\]

According to the proposition 1 from Blanchard and Kahn (1980), the solution of the ALM for inflation takes the following form:

\[
\pi_t = c_{\pi}^g a_t + \Omega_{\pi}^g \bar{x} + d_{\pi}^g e_t. \quad (A.17)
\]

We obtain using (6) and (A.17):

\[
E_t \pi_{t+1} = c_{\pi}^g E_t a_{t+1} + \Omega_{\pi}^g \bar{x}. \quad (A.18)
\]
Using (A.12) and (A.18) to eliminate $E_t \pi_{t+1}$ and arranging the terms yield:

\[ \pi_t = \frac{[A_{12} - c_{\pi}^g(1 - \gamma)]}{(c_{\pi}^g \gamma - A_{11})} a_t + \frac{(A_{13} - \Omega_{\pi}^g)}{(c_{\pi}^g \gamma - A_{11})} \bar{x} + \frac{P_{1,t}}{(c_{\pi}^g \gamma - A_{11})} e_t, \]  

(A.19)

This implies that:

\[ c_{\pi}^g = \frac{A_{12} - c_{\pi}^g(1 - \gamma)}{c_{\pi}^g \gamma - A_{11}}, \]  

(A.20)

\[ d_{\pi}^g = \frac{P_{1}}{c_{\pi}^g \gamma - A_{11}}, \]  

(A.21)

and

\[ \Omega_{\pi}^g = \frac{(A_{13} - \Omega_{\pi}^g)}{(c_{\pi}^g \gamma - A_{11})}. \]  

(A.22)

We gather (6) and (A.12), while using (A.10) to substitute $x_t$ to obtain:

\[ E_t y_{t+1} = Z + A_1 y_t + P_1 e_t, \]

where

\[ y_t \equiv [\pi_t, a_t], \quad A \equiv \begin{bmatrix} A_{11} & A_{12} \\ \gamma & 1 - \gamma \end{bmatrix}, \quad P \equiv \begin{bmatrix} P_{1} \\ 0 \end{bmatrix} \quad \text{and} \quad Z \equiv \begin{bmatrix} \Omega_{\pi}^g \bar{x} \\ 0 \end{bmatrix}. \]

The above system is subject to two boundary conditions: $a_0$ and $\lim_{s \rightarrow \infty} |E_t \pi_{t+s}| < \infty$. The eigenvalues of $A$ are $1 - \gamma$ and the two eigenvalues of $A_1$:

\[ A_1 = \begin{bmatrix} A_{11} & A_{12} \\ \gamma & 1 - \gamma \end{bmatrix}. \]  

(A.23)

We can show that $A_1$ has an eigenvalue inside and one outside the unit circle. Among infinite stochastic sequences of $c_{\pi}^g$ satisfying (A.20), we focus on a non-explosive solution, i.e., the solution corresponding to the eigenvalue of $A_1$ inside the unit circle.

It is straightforward to show that the trace and determinant of $A_1$ are both positive.
Thus, for $A_1$ to have two real eigenvalues $(\varphi_1, \varphi_2)$, one inside and one outside the unit circle, it is sufficient to show that $(1 - \varphi_1)(1 - \varphi_2) < 0$. This is equivalent to:

$$\mu_1 + \mu_2 > 1 + \mu_1 \mu_2.$$  \hfill (A.24)

Knowing that $\varphi_1 + \varphi_2$ is equal to the trace of $A_1$ and $\varphi_1 \varphi_2$ equal to its determinant, we rewrite (A.24) as:

$$A_{11} + 1 - \gamma > 1 + A_{11}(1 - \gamma) - A_{12}\gamma.$$

After simplification, we get:

$$\kappa^2 (\mu \phi + \delta_G) \left[1 - \beta (1 - \gamma)\right] + \alpha (1 - \beta) \left[1 - \beta [1 - \gamma (1 - \beta)]\right] > 0$$

which is always verified given that $\beta \in (0, 1)$, $\mu \in [0, 1]$, $\alpha \in [0, 1]$, $\phi \in [0, 1]$ and $\gamma \in (0, 1)$.

Rewriting (A.20) as $\epsilon_{\pi}^g \epsilon_{\pi}^g \gamma + \epsilon_{\pi}^g [(1 - \gamma) - A_{11}] - A_{12} = 0$ and substituting $A_{11}$ and $A_{12}$ by their expression, we determine the coefficients in

$$p_2 (\epsilon_{\pi}^g)^2 + p_1 \epsilon_{\pi}^g + p_0 = 0,$$  \hfill (A.25)

where

$$
\begin{align*}
 p_0 &= \alpha \beta \left\{1 - \beta (1 - \gamma) [1 - \gamma (1 - \beta)]\right\} > 0, \\
 p_1 &= \beta (1 - \gamma) \left\{\kappa^2 (\mu \phi + \delta_G) (1 - \gamma) + \alpha [1 - \gamma (1 - \beta)]\right\} \\
 &\quad - \kappa^2 (\mu \phi + \delta_G) - \alpha \left\{1 + \beta^2 \gamma [1 - \gamma (1 - \beta)]\right\}, \\
 p_2 &= \gamma \beta \left\{\kappa^2 (\mu \phi + \delta_G) (1 - \gamma) + \alpha [1 - \gamma (1 - \beta)]\right\} > 0
\end{align*}
$$

We rewrite $p_1$ as $p_1 = -\kappa^2 (\mu \phi + \delta_G) [1 - \beta (1 - \gamma)] - \alpha (1 - \beta) \left\{1 - \beta [1 - \gamma (1 - \beta)]\right\} - p_0 - p_2$, it follows immediately that $p_1 < 0$. Then, it is straightforward to show that the discriminant of the polynomial (A.25) is positive. To characterize the two solutions
of \( c^g_\pi \), we rewrite (A.25) as:

\[
\begin{align*}
  c^g_\pi &= -\frac{p_0 + p_2 \left(c^g_\pi\right)^2}{p_1} \equiv f(c^g_\pi) \\
  &\quad \text{(A.26)}
\end{align*}
\]

As \( f(c^g_\pi) \) is strictly increasing for \( c^g_\pi \in [0, 1] \) with \( f'(c^g_\pi) = -\frac{2p_2}{p_1} c^g_\pi > 0 \). To prove \( f(c^g_\pi) : [0, 1] \to (0, 1) \), it is sufficient to show that \( f(0) > 0 \) and that \( f(1) < 1 \). Then, it is immediate that \( f(0) = -\frac{p_0}{p_1} > 0 \) and that \( f(1) = -\frac{p_0 + p_2}{p_1} < 1 \) since

\[
p_1 = -\kappa^2 \left( \mu \phi + \delta G \right) \left[ 1 - \beta (1 - \gamma) \right] - \alpha \left( 1 - \beta \right) \left[ 1 - \beta \left( 1 - \gamma \right) \right] - p_0 - p_2.
\]

Knowing that \( f(c^g_\pi) : [0, 1] \to (0, 1) \) and \( f(c^g_\pi) \) is strictly increasing, it follows from the theorem of Brouwer that there exists one unique solution of \( c^g_\pi \) in the interval \((0, 1)\). This solution corresponds to

\[
  c^g_\pi = -\frac{p_1 - \sqrt{p_1^2 - 4p_2p_0}}{2p_2} \\
  &\quad \text{(A.27)}
\]

The other possible solution \( c^g_\pi = \frac{-p_1 + \sqrt{p_1^2 - 4p_2p_0}}{2p_2} \) is larger than unit. Therefore, the latter is rejected to avoid an explosive evolution of inflation.

### A.3 Properties of the feedback coefficients

We now show that \( f(c^g_\pi) : [0; \frac{\alpha^\beta}{\alpha + \kappa^2 (\mu \phi + \delta G)}] \to (0; \frac{\alpha^\beta}{\alpha + \kappa^2 (\mu \phi + \delta G)}) \). Knowing that \( f(0) > 0 \) and substituting \( c^g_\pi \) by \( \frac{\alpha^\beta}{\alpha + \kappa^2 (\mu \phi + \delta G)} \), into the function \( f(c^g_\pi) \) defined by (A.26), we find

\[
f\left( \frac{\alpha^\beta}{\alpha + \kappa^2 (\mu \phi + \delta G)} \right) = \frac{\alpha^\beta}{\alpha + \kappa^2 (\mu \phi + \delta G)} \left[ \frac{\alpha + \kappa^2 (\mu \phi + \delta G)}{\alpha \beta} p_0 + \frac{\alpha + \kappa^2 (\mu \phi + \delta G)}{\alpha \beta} p_2 \right].
\]

Using \( p_2 = \frac{\alpha (1 - \beta) + \kappa^2 (\mu \phi + \delta G)}{\alpha + \kappa^2 (\mu \phi + \delta G)} \) \( \frac{\alpha^\beta}{\alpha + \kappa^2 (\mu \phi + \delta G)} p_2 \), \( p_0 = \frac{\alpha (1 - \beta) + \kappa^2 (\mu \phi + \delta G)}{\alpha \beta^2} \frac{\alpha^\beta}{\alpha + \kappa^2 (\mu \phi + \delta G)} p_0 + \frac{\alpha^2 + \kappa^2 (\mu \phi + \delta G)}{\alpha^2 \beta} \frac{\alpha^\beta}{\alpha + \kappa^2 (\mu \phi + \delta G)} p_0 \)
and the definition of \( p_0, p_1, \) and \( p_2 \) given above, we rewrite the denominator as

\[
-p_1 = \beta p_2 + \frac{\alpha^2 + \kappa^2 (\mu \phi + \delta G)}{\alpha^2 \beta} p_0.
\]  
(A.29)

Substituting the above expression of \(-p_1\) into (A.28), we obtain:

\[
f\left(\frac{\alpha \beta}{\alpha + \kappa^2 (\mu \phi + \delta G)}\right) = \frac{\alpha \beta}{\alpha + \kappa^2 (\mu \phi + \delta G)} \left\{ \frac{\alpha^2 + \kappa^2 (\mu \phi + \delta G)}{\alpha^2 \beta} p_0 + \frac{\alpha + \kappa^2 (\mu \phi + \delta G)}{\alpha^2 \beta} p_2 \right\}
\]

Given that \( f'(c^g_\pi) = -\frac{2p_2}{p_1} c^g_\pi > 0 \) for \( c^g_\pi \in [0, 1] \), \( f(c^g_\pi) \) is strictly increasing in the interval \([0; \frac{\alpha \beta}{\alpha + \kappa^2 (\mu \phi + \delta G)}]\). This property and the fact that \( f(c^g_\pi) : [0; \frac{\alpha \beta}{\alpha + \kappa^2 (\mu \phi + \delta G)}] \to (0; \frac{\alpha \beta}{\alpha + \kappa^2 (\mu \phi + \delta G)}) \) imply that there is a unique solution for \( c^g_\pi \) so that \( 0 < c^g_\pi < \frac{\alpha \beta}{\alpha + \kappa^2 (\mu \phi + \delta G)} \).

**The case where \( \gamma = 0 \).** Substituting \( \gamma = 0 \) into (A.13)-(A.16) and using the results in (A.20)-(A.22), we obtain:

\[
A_{11} \equiv \frac{1}{\beta},
\]

\[
A_{12} \equiv -\frac{\alpha (1 - \beta)}{\kappa^2 (\mu \phi + \delta G) + \alpha},
\]

\[
A_{13} \equiv -\frac{\alpha \kappa (1 - \beta)}{\beta \kappa^2 (\mu \phi + \delta G) + \alpha},
\]

\[
P_1 \equiv -\frac{\alpha}{\beta \kappa^2 (\mu \phi + \delta G) + \alpha},
\]

45
and

\[ c^g_\pi = \frac{\alpha \beta}{\alpha + \kappa^2 (\mu \phi + \delta G)} , \]
\[ d^g_\pi = \frac{\alpha}{\alpha + \kappa^2 (\mu \phi + \delta G)} , \]
\[ \Omega^g_\pi = \frac{\alpha \kappa}{\alpha + \kappa^2 (\mu \phi + \delta G)} . \]

The case where \( \gamma = 1 \). Inserting \( \gamma = 1 \) into (A.13)-(A.16) and similarly, using the results in (A.20)-(A.22) lead to:

\[ A_{11} \equiv \frac{\kappa^2 (\mu \phi + \delta G) + \alpha (1 + \beta^3)}{\alpha \beta^2} , \]
\[ A_{12} \equiv -\frac{1}{\beta} , \]
\[ A_{13} \equiv -\frac{\alpha \kappa (1 - \beta^2)}{\alpha \beta^2} , \]
\[ P_1 \equiv -\frac{1}{\beta^2} , \]

and

\[ c^g_\pi = \frac{\kappa^2 (\mu \phi + \delta G) + \alpha (1 + \beta^3) - \sqrt{\left[ \kappa^2 (\mu \phi + \delta G) + \alpha (1 + \beta^3) \right]^2 - 4\alpha^2 \beta^2}}{2\alpha \beta^2} , \]
\[ d^g_\pi = \frac{\alpha}{\kappa^2 (\mu \phi + \delta G) + \alpha \left[ 1 + \beta^2 (\beta - c^g_\pi) \right] + \alpha \kappa (1 - \beta^2)} , \]
\[ \Omega^g_\pi = \frac{\alpha \kappa (1 - \beta^2)}{\kappa^2 (\mu \phi + \delta G) + \alpha \left[ 1 - \beta^2 (1 + c^g_\pi - \beta) \right]} . \]

A.4 The ALMs under constant gain learning

We rewrite the final form of the ALM for inflation:

\[ \pi_t = c^g_\pi a_t + \Omega^g_\pi x + d^g_\pi e_t . \]
where

\[
\begin{align*}
\alpha \pi' &= -p_1 - \sqrt{p_1^2 - 4p_2 p_0} \quad \text{(A.31)} \\
\beta \pi' &= \frac{\kappa^2 (\mu \phi + \delta G) (1 - c_{\pi}^g \gamma \beta) + \alpha \{1 + \beta \gamma (\beta - c_{\pi}^g) [1 - \gamma (1 - \beta)]\}}{\Phi(y)}, \quad \text{(A.32)} \\
\gamma \pi' &= \frac{\alpha \kappa \{1 - \beta [1 - \gamma (1 - \beta)]\}}{\Phi(y)}. \quad \text{(A.33)}
\end{align*}
\]

with \( \Phi(y) = \kappa^2 (\mu \phi + \delta G) + \alpha \{1 + \beta^2 \gamma [1 - \gamma (1 - \beta)]\} - \beta (c_{\pi}^g \gamma + 1) \{(1 - \gamma) [\kappa^2 (\mu \phi + \delta G) + \alpha] + \alpha \gamma \beta\} \). \quad \text{(A.34)}

Using the Phillips curve (1), (6) and (A.30), we obtain the ALM for the output gap:

\[
x_t = \frac{1}{\kappa} \left( c_{\pi}^g - \beta \right) a_t + \frac{\Omega_{\pi}^g}{\kappa} \bar{x} - \frac{(1 - d_{\pi}^g)}{\kappa} e_t
\]

that we can rewrite into

\[
x_t = c_{\pi}^g a_t + d_{\pi}^g e_t + \Omega_{\pi}^g \bar{x}, \quad \text{(A.35)}
\]

where \( c_{\pi}^g = -\frac{1}{\kappa} (\beta - c_{\pi}^g) \), \( d_{\pi}^g = -\frac{1}{\kappa} (1 - d_{\pi}^g) \), and \( \Omega_{\pi}^g = \frac{1}{\kappa} \Omega_{\pi}^g \).

Using (2) and (A.35), we get the ALM for the interest rate:

\[
r_t = c_r^g a_t + \delta_r^g b_t + d_r^g e_t + \Omega_r^g \bar{x}, \quad \text{(A.36)}
\]

with \( c_r^g = 1 + \frac{\gamma}{\kappa} (\beta - c_{\pi}^g) \), \( \delta_r^g = \sigma \), \( d_r^g = \frac{\gamma}{\kappa} (1 - d_{\pi}^g) \), and \( \Omega_r^g = -\frac{\gamma}{\kappa} \Omega_{\pi}^g \).
A.5 The effects of learning coefficient on feedback coefficients

Deriving $p_0$, $p_1$ and $p_2$ with respect to $\gamma$ and using (A.29), we get

$$
\frac{\partial p_2}{\partial \gamma} = \beta \kappa^2 \left( \mu \phi + \delta^G \right) (1 - 2\gamma) + \beta \alpha [1 - 2\gamma (1 - \beta)]
$$

$$
\frac{\partial p_0}{\partial \gamma} = \alpha \beta^2 [1 - \gamma (1 - \beta)] + \alpha \beta^2 (1 - \gamma) (1 - \beta) > 0,
$$

$$
\frac{\partial p_1}{\partial \gamma} = -\beta \frac{\partial p_2}{\partial \gamma} - \frac{\alpha + \kappa^2 (\mu \phi + \delta^G)}{\alpha \beta} \frac{\partial p_0}{\partial \gamma}.
$$

Alternatively, using the original expression of $p_1$, we obtain

$$
\frac{\partial p_1}{\partial \gamma} = -2 \beta \kappa^2 (\mu \phi + \delta^G) (1 - \gamma) - 2\alpha \beta [1 - \gamma (1 - \beta^2)].
$$

Deriving the solution of $c^g_{\pi}$ given by (A.27) yields:

$$
\frac{\partial c^g_{\pi}}{\partial \gamma} = \frac{1}{2p_2^2} \left( F \frac{\partial p_1}{\partial \gamma} + G \frac{\partial p_0}{\partial \gamma} \right),
$$

where

$$
F = -p_2 + \frac{-p_1 p_2}{\sqrt{p_1^2 - 4p_2 p_0}} - \frac{1}{\beta} p_1 - \frac{1}{\beta} \frac{p_1^2 - 2p_2 p_0}{\sqrt{p_1^2 - 4p_2 p_0}}.
$$

$$
G = \frac{2p_2 p_0}{\sqrt{p_1^2 - 4p_2 p_0}} \left( p_1 + \frac{p_1^2 - 2p_2 p_0}{\sqrt{p_1^2 - 4p_2 p_0}} \right) \frac{\alpha + \kappa^2 (\mu \phi + \delta^G)}{\beta \alpha}.
$$

Substituting $\frac{\partial p_1}{\partial \gamma}$ by its expression, using $p_1 = -\beta p_2 - \frac{\alpha + \kappa^2 (\mu \phi + \delta^G)}{\alpha \beta} p_0$, $p_2 = \frac{1}{\beta} [-p_1 - \frac{\alpha + \kappa^2 (\mu \phi + \delta^G)}{\alpha \beta} p_0]$, and $\frac{\partial p_2}{\partial \gamma} = \frac{1}{\beta} [\frac{\partial p_1}{\partial \gamma} - \frac{\alpha + \kappa^2 (\mu \phi + \delta^G)}{\alpha \beta} \frac{\partial p_0}{\partial \gamma}]$, factorizing by $p_1$ we get the following expression after tedious calculation (more details are available in the technical appendix):

$$
\frac{\partial c^g_{\pi}}{\partial \gamma} = \frac{1}{2p_2^2} \left( F \frac{\partial p_1}{\partial \gamma} + G \frac{\partial p_0}{\partial \gamma} \right),
$$

where

Using $p_1 = -\beta p_2 - \frac{\alpha + \kappa^2 (\mu \phi + \delta^G)}{\alpha \beta} p_0$, after fastidious arrangements of terms, we finally
obtain:
\[
\frac{\partial c_g}{\partial \gamma} = \frac{1 - \frac{\alpha + \kappa^2 (\mu \phi + \delta^G) c_g}{\alpha \beta}}{\beta p_2 \sqrt{p_1^2 - 4 p_2 p_0}} \left( p_0 \frac{\partial p_1}{\partial \gamma} - p_1 \frac{\partial p_0}{\partial \gamma} \right).
\]

Using \(0 < c_g < \frac{\alpha \beta}{\alpha + \kappa^2 (\mu \phi + \delta^G)}\), we obtain: \(1 - \frac{\alpha + \kappa^2 (\mu \phi + \delta^G) c_g}{\alpha \beta} > 0\). To determine the sign of \(H \equiv p_0 \frac{\partial p_1}{\partial \gamma} - p_1 \frac{\partial p_0}{\partial \gamma}\), we first check its value for and then its derivative with respect to \(\gamma\). For \(\gamma = 1\), we have: \(\frac{\partial p_0}{\partial \gamma} = \beta^3 \alpha > 0\), \(\frac{\partial p_1}{\partial \gamma} = -2 \beta^3 \alpha < 0 = \beta \frac{\partial p_2}{\partial \gamma} + \frac{\alpha + \kappa^2 (\mu \phi + \delta^G)}{\alpha \beta} \frac{\partial p_0}{\partial \gamma}\), \(p_1 = -(1 + \beta^3) \alpha - \kappa^2 (\mu \phi + \delta^G)\) and \(p_0 = \beta \alpha\).

The results in the above for \(\gamma = 1\) yield:
\[
H \equiv p_0 \frac{\partial p_1}{\partial \gamma} - p_1 \frac{\partial p_0}{\partial \gamma} = \beta^3 \alpha \left[ (1 + \beta^3) \alpha + \kappa^2 (\mu \phi + \delta^G) - 2 \beta \alpha \right] < 0
\]
if only and only if
\[
\mu \phi < -\frac{(1 + \beta^3 - 2 \beta) \alpha}{\kappa^2} - \delta^G. \tag{A.37}
\]

In the case where \(\phi = 0\) and/or \(\mu = 0\), it is always verified. When \(\phi = 1\), \(\mu\) must respect the following condition: \(\mu < -\frac{(1 + \beta^3 - 2 \beta) \alpha}{\kappa^2} - \delta^G\). When \(\mu = 1\), \(\phi\) must respect the following condition \(\phi < -\frac{(1 + \beta^3 - 2 \beta) \alpha}{\kappa^2} - \delta^G\). These two conditions impose a limit for the degree of negociation (when \(\phi = 1\)) and conservatism (when \(\mu = 1\)) but this limit is high enough according to simulations. For the highest degree of conservatism, where \(\phi = 1\), then \(\mu < 0.804\).

Deriving \(H\) with respect to \(\gamma\) yields
\[
\frac{\partial H}{\partial \gamma} = \frac{\partial p_0}{\partial \gamma} \frac{\partial p_1}{\partial \gamma} + p_0 \frac{\partial^2 p_1}{\partial^2 \gamma} - \frac{\partial p_1}{\partial \gamma} \frac{\partial p_0}{\partial \gamma} - p_1 \frac{\partial^2 p_0}{\partial^2 \gamma} = p_0 \frac{\partial^2 p_1}{\partial^2 \gamma} - p_1 \frac{\partial^2 p_0}{\partial^2 \gamma}
\]
Deriving twice $p_0$ and $p_1$ with respect to $\gamma$, $\forall \gamma \in (0,1)$, leads to

\[
\frac{\partial^2 p_2}{\partial^2 \gamma} = -2\beta \left\{ \kappa^2 (\mu \phi + \delta^G) + \alpha (1 - \beta) \right\} < 0
\]

\[
\frac{\partial^2 p_0}{\partial^2 \gamma} = -2\alpha \beta^2 (1 - \beta) < 0,
\]

\[
\frac{\partial^2 p_1}{\partial^2 \gamma} = 2\beta \left\{ \kappa^2 (\mu \phi + \delta^G) + \alpha (1 - \beta^2) \right\}
= -\beta \frac{\partial^2 p_2}{\partial^2 \gamma} - \frac{\alpha + \kappa^2 (\mu \phi + \delta^G) c_{\phi^G}}{\alpha \beta} \frac{\partial^2 p_0}{\partial^2 \gamma}
\]

Replacing those values into $\frac{\partial H}{\partial \gamma}$, it yields: $\frac{\partial H}{\partial \gamma} > 0$. Consequently, given that $H < 0$ for $\gamma = 1$ and $\frac{\partial H}{\partial \gamma} > 0$, $\forall \gamma \in (0,1)$, we conclude that:

\[
\frac{\partial c^g}{\partial \gamma} < 0.
\]

Using $d^g_\pi$ with respect to $\gamma$ yields:

\[
\frac{\partial d^g_\pi}{\partial \gamma} = -\alpha \beta \frac{2\gamma \alpha \beta (\beta - c^g_\phi) + (1 - 2\gamma) \left\{ \alpha \beta - [\alpha + \kappa^2 (\mu \phi + \delta^G)] c^g_\phi \right\} - \gamma \left[ (1 - \gamma) \left\{ \alpha + \kappa^2 (\mu \phi + \delta^G) \right\} + \gamma \alpha \beta \frac{\partial c^g_\phi}{\partial \gamma} \right\} \kappa^2 (\mu \phi + \delta^G) + \alpha \gamma \alpha \beta^2 (\beta - c^g_\phi) + \beta \gamma (1 - \gamma) \left\{ \alpha \beta - [\alpha + \kappa^2 (\mu \phi + \delta^G)] c^g_\phi \right\}}{\alpha + \kappa^2 (\mu \phi + \delta^G)}.
\]

Using $c^g_\pi < \frac{\alpha \beta}{\alpha + \kappa^2 (\mu \phi + \delta^G)}$, we find that:

\[
2\gamma \alpha \beta (\beta - c^g_\pi) + (1 - 2\gamma) \left\{ \alpha \beta - \kappa^2 (\mu \phi + \delta^G) \right\} c^g_\pi \right\} > 0,
\]

it follows that:

\[
\frac{\partial d^g_\pi}{\partial \gamma} < 0.
\]

Using the definition of $c^g_\pi$, $d^g_\pi$, $c^g_\pi$ and $d^g_\pi$, it is straightforward to show the sign of their partial derivative with respect to $\gamma$.

We want to show that $\frac{\partial c^g_\pi}{\partial \gamma} < 0$. We first differentiate $\Phi(y)$ with respect to $\gamma$ which
gives:

$$
\Phi'(y) = \alpha \beta \left\{ (\beta - c'^{\pi}_{\pi}) [1 - 2\gamma (1 - \beta)] + (1 - \beta) - \beta \frac{\partial c'^{\pi}_{\pi}}{\partial \gamma} \right\} \\
- \beta \gamma (1 - \gamma) \frac{\partial c'^{\pi}_{\pi}}{\partial \gamma} [\alpha + \kappa^2 (\mu \phi + \delta^G)] - \beta \kappa^2 (\mu \phi + \delta^G) (c'^{\pi}_{\pi} (1 - 2\gamma) - 1)
$$

$$
\frac{\partial \Omega^{c^g}_{\pi}}{\partial \gamma} = \alpha \kappa \beta (1 - \beta) \Phi(y) - \Phi'(y) \left\{ 1 - \beta [1 - \gamma (1 - \beta)] \right\} \Phi^2(y)
$$

(A.38)

Using $0 < c^g_{\pi} < \frac{\alpha \beta}{\alpha + \kappa^2 (\mu \phi + \delta^G)}$, we show that (A.38) is negative. Since we have an upper bound, we demonstrate that this expression is negative by reduction to the absurd in the technical appendix. Therefore,

$$
\frac{\partial \Omega^{c^g}_{\pi}}{\partial \gamma} < 0. \square
$$

A.6 The effect of negotiation on feedback coefficients

Derivating $p_0$, $p_1$, and $p_2$ with respect to $\mu$ yields:

$$
\frac{\partial p_0}{\partial \mu} = 0\\
\frac{\partial p_2}{\partial \mu} = \gamma \beta \kappa^2 (\phi + \delta^G) (1 - \gamma) > 0\\
\frac{\partial p_1}{\partial \mu} = -\gamma \beta^2 \kappa^2 (\phi + \delta^G) (1 - \gamma) < 0
$$
Here we want to show that $\frac{\partial c^g}{\partial \mu} < 0$.

$$\frac{\partial c^g}{\partial \mu} = \left( -\frac{\partial p_1}{\partial \mu} - \frac{2p_1 \frac{\partial p_1}{\partial \mu} - 4p_2 \frac{\partial p_0}{\partial \mu} - 4p_0 \frac{\partial p_1}{\partial \mu}}{2\sqrt{p_1^2 - 4p_2 p_0}} \right) p_2 - \left( -p_1 - \sqrt{p_1^2 - 4p_2 p_0} \right) \frac{\partial p_2}{\partial \mu} \Rightarrow$$

After fastidious calculus (see the technical appendix for more details), we obtain the following expression:

$$\frac{\partial c^g}{\partial \mu} = \frac{1}{2p_2^2} F \frac{\partial p_1}{\partial \mu}$$

where $F = -p_2 + \frac{-p_1 p_2}{\sqrt{p_1^2 - 4p_2 p_0}} - \frac{1}{\beta} p_1 - \frac{1}{\beta} \frac{p_1^2 - 2p_2 p_0}{\sqrt{p_1^2 - 4p_2 p_0}}$.

Using $p_1 = -\beta p_2 - \frac{\alpha + \kappa^2 (\mu \phi + \delta G)}{\alpha \beta} p_0$, after arrangements of terms, we finally obtain:

$$\frac{\partial c^g}{\partial \mu} = \frac{1 - \frac{\alpha + \kappa^2 (\mu \phi + \delta G)}{\alpha \beta} c^g_\pi}{\beta p_2 \sqrt{p_1^2 - 4p_2 p_0}} p_0 \frac{\partial p_1}{\partial \mu}.$$

Using $0 < c^g_\pi < \frac{\alpha \beta}{\alpha + \kappa^2 (\mu \phi + \delta G)}$, we obtain: $1 - \frac{\alpha + \kappa^2 (\mu \phi + \delta G)}{\alpha \beta} c^g_\pi > 0$. To determine the sign of $H \equiv p_0 \frac{\partial p_1}{\partial \mu}$, we first check its value for and then its derivative with respect to $\mu$. For any $\mu$, we have:

$$H = p_0 \frac{\partial p_1}{\partial \mu} < 0$$

Consequently, given that $H < 0$ for $\gamma = 1$ and $\frac{\partial H}{\partial \mu} = 0$, $\forall \gamma \in [0, 1]$, we conclude that

$$\frac{\partial c^g}{\partial \mu} < 0.$$

Derivating $d^g_\pi$ with respect to $\mu$ yields:

$$\frac{\partial d^g_\pi}{\partial \mu} = -\alpha \left\{ \kappa^2 \phi (1 - c^g_\pi \gamma \beta) - \kappa^2 \gamma \beta \frac{\partial c^g_\pi}{\partial \mu} \left( \mu \phi + \delta G \right) - \frac{\partial c^g_\pi}{\partial \mu} \alpha \{ \beta \gamma [1 - \gamma (1 - \beta)] \} \right\} \left\{ \kappa^2 (\mu \phi + \delta G) (1 - c^g_\pi \gamma \beta) + \alpha + \alpha \beta \gamma (\beta - c^g_\pi) [1 - \gamma (1 - \beta)] \right\}^2$$

Knowing that $0 < c^g_\pi < \frac{\alpha \beta}{\alpha + \kappa^2 (\mu \phi + \delta G)}$, then we want to show that $(1 - c^g_\pi \gamma \beta) > 0$,
then we replace $c^g_\pi$ by its upper bound since $\frac{\partial c^g_\pi}{\partial \mu} < 0$. It gives:

$$
(1 - c^g_\pi \gamma/\beta) < \frac{\alpha(1-\beta^2\gamma)+\kappa^2(\mu \phi + \delta G)}{\alpha+\kappa^2(\mu \phi + \delta G)}$

where $\frac{\alpha(1-\beta^2\gamma)+\kappa^2(\mu \phi + \delta G)}{\alpha+\kappa^2(\mu \phi + \delta G)} > 0$. Using the above result and the fact that $\frac{\partial c^g_\pi}{\partial \mu} < 0$, it follows that:

$$
\frac{\partial c^g_\pi}{\partial \mu} < 0.
$$

We now look for the sign of $\frac{\partial \Omega^g_\pi}{\partial \mu}$. We define

$$
\Phi_\mu \equiv \kappa^2 (\mu \phi + \delta G) + \alpha \left\{ 1 + \beta^2 \gamma [1 - \gamma (1 - \beta)] \right\} - \beta (c^g_\pi \gamma + 1) \left\{ 1 - \gamma \right\} \left[ \kappa^2 (\mu \phi + \delta G) + \alpha \right] + \alpha \gamma \beta
$$

Differentiating $\Phi_\mu$ with respect to $\mu$ leads to:

$$
\Phi'_\mu \equiv \kappa^2 \phi [1 - \beta (1 - \gamma)] - \beta \kappa^2 (1 - \gamma) c^g_\pi \gamma \phi \\
- \beta \gamma \left[ \kappa^2 (1 - \gamma) (\delta G + \mu \phi) + \alpha (1 - \gamma) + \alpha \gamma \beta \right] \frac{\partial c^g_\pi}{\partial \mu}
$$

Consequently, we have:

$$
\frac{\partial \Omega^g_\pi}{\partial \mu} = \alpha \kappa \left\{ 1 - \beta [1 - \gamma (1 - \beta)] \right\} \left\{ \frac{-\kappa^2 \phi [1 - \beta (1 - \gamma)] + \beta \kappa^2 (1 - \gamma) c^g_\pi \gamma \phi}{\Phi_\mu^2} \right. \\
+ \left. \beta \gamma \left[ \kappa^2 (1 - \gamma) (\delta G + \mu \phi) + \alpha (1 - \gamma) + \alpha \gamma \beta \right] \frac{\partial c^g_\pi}{\partial \mu} \right\} \Phi_\mu^2
$$

(A.40)

We want to show that $\frac{\partial \Omega^g_\pi}{\partial \mu} < 0$. For this matter, we just need to study the sign of
\[-\kappa^2 \phi [1 - \beta (1 - \gamma)] + \beta \kappa^2 (1 - \gamma) \gamma \phi \bar{e}_\pi^G \text{ using the upper bound,} \]

\[\gamma \alpha \beta - [1 - \beta (1 - \gamma)] \left\{ \gamma \alpha \beta + \alpha + \kappa^2 (\mu \phi + \delta \gamma) \right\} < 0\]

Then,

\[\frac{\partial \Omega_{\pi}^G}{\partial \mu} < 0. \square\]

A.7 The optimal degree of instrument-independence under learning

The government signs a long-term contract with the central bank to improve the social welfare without accounting for the short-run issue of monetary policy, including the manipulation of private expectations to improve the short-run macroeconomic stabilization. It takes \(E_{t-1} \pi_t = E_t \pi_{t+1} = E_t a_t\), then inserting the latter into (28) gives \(E_t \pi_{t+1} = c^G_{\pi} E a_t + \Omega_{\pi}^G \tilde{x}\), that yields the expected inflation on the long-term approach:

\[E_{\pi t} = \frac{\Omega_{\pi}^G}{1 - c^G_{\pi}} \tilde{x}, \quad (A.41)\]

According to (32), and using \(E_{\pi t} = E a_t\) and (A.41), we obtain for the expected output gap:

Substituting \(c^G_{x} = -\beta - \frac{c^G_{\pi}}{\kappa}, \Theta^G_{x} = \frac{1}{\kappa} \Theta^G_{\pi}\) and \(\Omega^G_{x} = \frac{1}{\kappa} \Omega^G_{\pi}\) into the previous equation leads to

\[E_{x t} = \frac{\Omega^G_{\pi} (1 - \beta)}{\kappa (1 - c^G_{x})} \tilde{x}, \quad (A.42)\]

Substituting \(E_{\pi t}\) and \(E_{x t}\) given respectively by (A.41) and (A.42) into the government loss function (4) gives

\[L^G_{t} = \frac{1}{2} E \sum_{i=0}^{+\infty} \beta^i \left[ \delta^G \left( \frac{\Omega^G_{\pi}}{1 - c^G_{\pi}} \tilde{x} \right)^2 + \alpha \left( \frac{\Omega^G_{\pi} (1 - \beta)}{\kappa (1 - c^G_{\pi})} \tilde{x} - \tilde{x} \right)^2 \right], \]

54
\[ L_t^G = \frac{1}{2} E_t \sum_{i=0}^{+\infty} \beta^i \left[ \delta^G \kappa^2 (\Omega_{\pi}^{cg})^2 + \alpha (\Omega_{\pi}^{cg})^2 (1 - \beta)^2 - \alpha \kappa^2 (1 - c_{\pi}^{cg})^2 - 2 \alpha \kappa (1 - \beta) (1 - c_{\pi}^{cg}) \Omega_{\pi}^{cg} \right] \tilde{x}^2, \]

Under learning, the first-order condition of the government’s minimization problem with respect to the inflation penalty rate:

\[
\frac{\partial L_t^S}{\partial \mu} = 0 \Rightarrow \frac{\partial \Omega_{\pi}^{cg}}{\partial \mu} (1 - c_{\pi}^{cg})^2 \left[ \delta^G \kappa^2 + \alpha (1 - \beta)^2 - \alpha \kappa (1 - \beta) (1 - c_{\pi}^{cg}) \right]
\]
\[+ \Omega_{\pi}^{cg} \frac{\partial c_{\pi}^{cg}}{\partial \mu} \left\{ \Omega_{\pi}^{cg} \left[ \delta^G \kappa^2 + \alpha (1 - \beta)^2 \right] - \alpha \kappa (1 - \beta) \left[ 1 - (c_{\pi}^{cg})^2 \right] \right\} = 0 \]

B Appendix for decreasing-gain learning

B.1 The solution of the ALMs

Similarly to the method used for solving the ALMs for inflation, the output gap and the interest rate, we use intertemporal tradeoff condition obtained from (A.8), (A.10), and (A.11) into (A.8), it gives:

\[
\left\{ \frac{\kappa^2 (\mu \phi + \delta^G) + \alpha \beta^2 (1 - \gamma_{t+2})}{\gamma_{t+1}} + \beta \right\} \pi_t = \beta \left\{ \kappa^2 (\mu \phi + \delta^G) (1 - \gamma_{t+2}) + \alpha \left[ (1 - \gamma_{t+2}) + \beta \right] \right\} E_t \pi_{t+1}
\]
\[+ \frac{\alpha}{\gamma_{t+1}} \beta a_t - \alpha \beta^2 \left[ (1 - \gamma_{t+2}) + \beta \right] (1 - \gamma_{t+1}) a_t
\]
\[- \alpha \kappa \left[ \beta (1 - \gamma_{t+2}) + \beta^2 - \frac{1}{\gamma_{t+1}} \right] \tilde{x} + \frac{\alpha}{\gamma_{t+1}} e_t \]

We now rewrite the above expression using \( \gamma_t = \frac{1}{t} \), \( \gamma_{t+1} = \frac{1}{t+1} \), \( \gamma_{t+2} = \frac{1}{t+2} \) and arrange to obtain:

\[ E_t \pi_{t+1} = A_{11,t} \pi_t + A_{12,t} a_t + P_{1,t} e_t + A_{13,t} \tilde{x} \quad \text{(B.1)} \]

55
where

\begin{align}
A_{11,t} & \equiv \frac{[\kappa^2 (\mu \phi + \delta^G) + \alpha] + \alpha \beta^2 \left(\frac{1}{t+1}\right)^2 (t + 1 + \beta)}{\beta \left\{ \kappa^2 (\mu \phi + \delta^G) + \alpha \left[ 1 + \frac{\beta}{t+1} \right] \right\}}, \\
A_{12,t} & \equiv -\frac{\alpha \beta \left[ 1 - \beta \frac{t(t+1+\beta)}{(t+1)^2} \right]}{\beta \left\{ \kappa^2 (\mu \phi + \delta^G) + \alpha \left[ 1 + \frac{\beta}{t+1} \right] \right\}}, \\
P_{1,t} & \equiv -\frac{\alpha \left\{ \kappa^2 (\mu \phi + \delta^G) + \alpha \left[ 1 + \frac{\beta}{t+1} \right] \right\}}{\beta \left\{ \kappa^2 (\mu \phi + \delta^G) + \alpha \left[ 1 + \frac{\beta}{t+1} \right] \right\}}, \\
A_{13,t} & \equiv \frac{\alpha \kappa \left[ (1 - \beta) - \frac{\beta^2}{t+1} \right]}{\beta \left\{ \kappa^2 (\mu \phi + \delta^G) + \alpha \left[ 1 + \frac{\beta}{t+1} \right] \right\}}.
\end{align}

The solution of the ALM of inflation takes the following form:

\[ \pi_t = c_{\pi,t}^g a_t + d_{\pi,t}^g e_t + \Omega_{\pi t}. \]

Using (6) and (B.6), we obtain:

\[ E_t \pi_{t+1} = c_{\pi,t+1}^g \left[ (1 - \gamma_{t+1}) a_t + \gamma_{t+1} \pi_t \right] \]

Using equations (B.7) and (B.1) to eliminate \( E_t \pi_{t+1} \) and arranging the terms yield:

\[ \pi_t = -\frac{A_{12,t} - \left(1 - \frac{1}{t+1}\right)c_{\pi,t+1}^g}{\frac{1}{t+1}c_{\pi,t+1}^g - A_{11,t}} a_t + \frac{P_{1,t}}{\frac{1}{t+1}c_{\pi,t+1}^g - A_{11,t}} e_t + \frac{A_{13,t} - \Omega_{\pi,t+1}^g}{\frac{1}{t+1}c_{\pi,t+1}^g - A_{11,t}} \]

This implies that

\[ c_{\pi,t}^g = -\frac{A_{12,t} - \left(1 - \frac{1}{t+1}\right)c_{\pi,t+1}^g}{\frac{1}{t+1}c_{\pi,t+1}^g - A_{11,t}}, \]

\[ d_{\pi,t}^g = \frac{P_{1,t}}{\frac{1}{t+1}c_{\pi,t+1}^g - A_{11,t}}, \]

\[ \Omega_{\pi,t} = \frac{A_{13,t} - \Omega_{\pi,t+1}^g}{\frac{1}{t+1}c_{\pi,t+1}^g - A_{11,t}}. \]

We gather equations (6), and (B.1), while using (A.11) to substitute \( x_t \) to obtain
the system of three equations:

\[ E_t y_{t+1} = A_t y_t + B_t \bar{x} + P_t e_t \]

where

\[ y_t \equiv [\pi_t, a_t, b_t], \quad A_t \equiv \begin{bmatrix} A_{11} & A_{12} & 0 \\ \frac{1}{t+1} & \frac{t}{t+1} & 0 \\ \frac{1}{\kappa(t+1)} & -\frac{-\beta}{\kappa(t+1)} & \frac{t}{t+1} \end{bmatrix}, \quad B_t \equiv \begin{bmatrix} \Omega_t \\ 0 \end{bmatrix}, \quad \text{and} \quad P_t \equiv \begin{bmatrix} P_{1,t} \\ 0 \end{bmatrix}. \]

The above system is subject to three boundary conditions: \( a_0, b_0, \) and \( \lim_{s \to \infty} |E_t \pi_{t+s}| < \infty. \) The eigenvalues of \( A_t \) are given by \( \frac{t}{t+1} \) and by the two eigenvalues of \( A_{1,t} : \)

\[ A_{1,t} = \begin{bmatrix} A_{11} & A_{12} \\ \frac{1}{t+1} & \frac{t}{t+1} \end{bmatrix} \quad \text{(B.12)} \]

Finally, we obtain the final expression of the ALM for inflation:

\[ \pi_t = c_{\pi,t}^{dg} a_t + d_{\pi,t}^{dg} e_t + \Omega_t \bar{x}. \quad \text{(B.13)} \]

\[ c_{\pi,t}^{dg} = \frac{\beta \left\{ \alpha - (1 - \gamma_{t+1}) \left[ \alpha(1 + \gamma_{t+1}) \left( \beta - c_{\pi,t+1}^{dg} \right) - c_{\pi,t+1}^{dg} \kappa^2 \left( \mu \phi + \delta^G \right) \right] \right\}}{\alpha + \kappa^2 (\mu \phi + \delta^G) \left( 1 - \beta \gamma_{t+1} c_{\pi,t+1}^{dg} \right) + \alpha \beta \gamma_{t+1} (1 + \beta \gamma_{t+1}) \left( \beta - c_{\pi,t+1}^{dg} \right)}, \]

\[ d_{\pi,t}^{dg} = -\frac{\alpha}{\alpha + \kappa^2 (\mu \phi + \delta^G) \left( 1 - \beta \gamma_{t+1} c_{\pi,t+1}^{dg} \right) + \alpha \beta \gamma_{t+1} (1 + \beta \gamma_{t+1}) \left( \beta - c_{\pi,t+1}^{dg} \right)}, \]

and

\[ \Omega_t = \frac{-\alpha \kappa \left[ (1 - \beta) - \gamma_{t+1} \beta^2 \right] + \Omega_{\pi,t+1}^{dg} \beta \left\{ \kappa^2 (\mu \phi + \delta^G) + \alpha [1 + \beta \gamma_{t+1}] \right\}}{\kappa^2 (\mu \phi + \delta^G) \left( 1 - \beta \gamma_{t+1} c_{\pi,t+1}^{dg} \right) + \alpha \left\{ 1 + \beta \gamma_{t+1} (1 + \beta \gamma_{t+1}) \left( \beta - c_{\pi,t+1}^{dg} \right) \right\}}. \]
Similarly, using the Phillips curve (1) and (B.13) yields the ALM for the output gap is:

$$x_t = c_x^d a_t + d_x^d e_t + \Omega_x^d \bar{x},$$  \hspace{1cm} (B.14)$$

where $c_x^d = -\frac{1}{\kappa} \left( \beta - c_{\pi,t}^d \right)$, $d_x^d = -\frac{1}{\kappa} \left( 1 - d_{\pi,t}^d \right)$, and $\Omega_x^d = \frac{1}{\kappa} \Omega_{\pi,t}^d$.

Using (B.14) and (2), and the definition of expected inflation and output gap (6)-(7) with decreasing gain yields the ALM for the interest rate:

$$r_t = c_r^d a_t + \delta_r^d b_t + d_r^d e_t + \Omega_r^d \bar{x},$$  \hspace{1cm} (B.15)$$

with $c_r^d = 1 + \frac{\sigma}{\kappa} \left( \beta - c_{\pi,t}^d \right)$, $\delta_r^d = \frac{\sigma}{\kappa}$, $d_r^d = \frac{\sigma}{\kappa} \left( 1 - d_{\pi,t}^d \right)$, and $\Omega_r^d = -\frac{\sigma}{\kappa} \Omega_{\pi,t}^d$. Since $0 < c_{\pi,t}^d < \alpha \beta \left( \alpha + \kappa^2 \left( \mu \phi + \delta^G \right) \right)$, $(\beta - c_{\pi,t}^d) > 0$ and the interest-rate rule is active since $c_r^d > 1$.

**B.2 The single stable solution under decreasing-gain learning**

We look for a non-explosive solution among infinite stochastic sequences that are satisfying equation (B.9). To characterize the properties of this solution, the value of $c_{\pi,t}^d$ is examined when $t \to +\infty$. Using the boundary conditions $\lim_{t \to +\infty} A_{11,t} = \frac{1}{\beta}$, and $\lim_{t \to +\infty} A_{12,t} = -\frac{\alpha \beta (1-\beta)}{\beta \alpha \kappa^2 (\mu \phi + \delta^G) + \alpha}$, we find that in the limit, $c_{\pi,t}^d$ evolves according to:

$$\lim_{t \to +\infty} c_{\pi,t}^d = \beta \lim_{t \to +\infty} \left[ -c_{\pi,t+1}^d - \frac{\alpha \beta (1-\beta)}{\alpha + \kappa^2 (\mu \phi + \delta^G)} \right]$$  \hspace{1cm} (B.16)$$

The boundary condition $\lim_{n \to +\infty} |\pi_{t+n}| < \infty$ implies that $\lim_{n \to +\infty} \beta^n c_{\pi,t+n}^d = 0$. Using this condition and solving (B.16) forward yield one and only one bounded solution of $c_{\pi,t}^d$:

$$\lim_{t \to +\infty} c_{\pi,t}^d = \frac{\alpha \beta (1-\beta)}{\alpha + \kappa^2 (\mu \phi + \delta^G)}.$$

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Furthermore, it follows from (B.16) that

\[
\lim_{t \to +\infty} c_{\pi,t+1}^{dg} \geq \lim_{t \to +\infty} c_{\pi,t}^{dg},
\]

implying that when \( t \to +\infty \), we have \( c_{\pi,t}^{dg} < \frac{\alpha \beta (1 - \beta)}{[\alpha + \kappa^2 (\mu \phi + \delta G)]} \). Assuming that for \( t = n + 1 \), we have \( c_{\pi,n+1}^{dg} < \frac{\alpha \beta (1 - \beta)}{[\alpha + \kappa^2 (\mu \phi + \delta G)]} \). It follows from (B.9) that

\[
c_{\pi,n}^{dg} = \frac{A_{12,n} - n+1 c_{\pi,n+1}^{dg}}{n c_{\pi,n}^{dg} - A_{11,n}}.
\]

Substituting \( A_{12,n} \) and \( A_{12,n} \) by their respective expression given by (B.2)-(B.3), we obtain:

\[
A_{11,n} \equiv \frac{[\kappa^2 (\mu \phi + \delta G) + \alpha] + \alpha \beta^2 \left( \frac{1}{n+1} \right) \left[ 1 + \left( \frac{1}{n+1} \right) \beta \right]}{\beta \left[ \kappa^2 (\mu \phi + \delta G) + \alpha \left( 1 + \beta \frac{1}{n+1} \right) \right]},
\]

\[
A_{12,n} \equiv \frac{-\alpha \beta - \alpha^2 \beta^2 (1 - \frac{1}{n+1}) (1 + \frac{1}{n+1} \beta)}{\beta \left[ \kappa^2 (\mu \phi + \delta G) + \alpha \left[ 1 + \beta \frac{1}{n+1} \right] \right]},
\]

We go back to the limit calculation:

\[
\left\{ A_{11,n} \left[ \alpha + \kappa^2 (\mu \phi + \delta G) \right] - \frac{\alpha \beta (1 - \beta)}{n+1} \right\} c_{\pi,n}^{dg} < \frac{\alpha \beta (1 - \beta) n}{n+1} - A_{12,n} \left[ \alpha + \kappa^2 (\mu \phi + \delta G) \right]
\]

\[
\Rightarrow c_{\pi,n}^{dg} < \frac{n}{n+1} \alpha \beta (1 - \beta) + \frac{\alpha \beta - \alpha^2 \beta^2 (1 - \frac{1}{n+1}) (1 + \frac{1}{n+1} \beta)}{\beta \left[ \kappa^2 (\mu \phi + \delta G) + \alpha \left[ 1 + \beta \frac{1}{n+1} \right] \right]} \left[ \alpha + \kappa^2 (\mu \phi + \delta G) \right] - \frac{\alpha \beta (1 - \beta)}{n+1} \Rightarrow
\]

\[
\Rightarrow c_{\pi,n}^{dg} < \frac{\Psi}{\Xi},
\]

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where

\[
\Psi = \frac{n}{n+1} \alpha \beta (1 - \beta) \beta \left\{ \kappa^2 \left( \mu \phi + \delta^G \right) + \alpha \left[ 1 + \beta \frac{1}{n+1} \right] \right\} \\
+ \left\{ \alpha \beta - \alpha \beta^2 \frac{(1 - \frac{1}{n+1})(1 + \frac{1}{n+1} \beta)}{1-n} \right\} \left[ \alpha + \kappa^2 \left( \mu \phi + \delta^G \right) \right],
\]

\[
\Xi = \left[ \alpha + \kappa^2 \left( \mu \phi + \delta^G \right) \right] \left\{ \frac{1}{n+1} \left[ \kappa^2 \left( \mu \phi + \delta^G \right) + \alpha \right] \right\}
\]
\[
- \frac{1}{n+1} \alpha \beta^2 \left( 1 - \beta \right) \left[ \kappa^2 \left( \mu \phi + \delta^G \right) + \alpha \left[ 1 + \beta \frac{1}{n+1} \right] \right].
\]

We rewrite both expressions as:

\[
\Psi = \frac{1}{n+1} \left\{ \alpha \beta^2 \left[ \alpha + \kappa^2 \left( \mu \phi + \delta^G \right) \right] \frac{n(1 - \beta) - \beta}{1-n} + \beta n \alpha^2 \beta^2 \frac{(1 - \beta)}{n+1} + \frac{\alpha \beta^2 [\alpha + \kappa^2 \left( \mu \phi + \delta^G \right)] \left( 1 + \beta \frac{1}{n+1} \right)}{(n+1)} \right\},
\]

\[
\Xi = \left[ \kappa^2 \left( \mu \phi + \delta^G \right) + \alpha \right]^2 + \alpha \beta^2 \left( \frac{1}{n+1} \right) \left[ \kappa^2 \left( \mu \phi + \delta^G \right) + \alpha \right] \left[ 1 + \beta \left( \frac{1}{n+1} \right) \right]
\]
\[
- \alpha \beta^2 \left( \frac{1}{n+1} \right) \left[ \kappa^2 \left( \mu \phi + \delta^G \right) + \alpha + \frac{1}{n+1} \alpha \beta \right].
\]

Replacing \( \Psi \) and \( \Xi \) yield after some rearrangements:

\[
c_{dg}^{\pi,n} < \frac{\alpha \beta \left[ 1 - \frac{\beta n}{(n+1)} \left( 1 + \frac{1}{n+1} \beta \right) \right]}{\left[ \alpha + \kappa^2 \left( \mu \phi + \delta^G \right) + \frac{\alpha \beta^2}{(n+1)} \left( 1 + \frac{1}{n+1} \beta \right) \right]}.\]

Thus, by taking the limit when \( n \to +\infty \), we can deduce that for \( t \in [1, +\infty[ \),

\[
c_{dg}^{\pi,t} < \frac{\alpha \beta}{\alpha + \kappa^2 \left( \mu \phi + \delta^G \right)}.
\]
C The linear inflation contract

C.1 Benchmark under RE: optimal inflation penalty

Solving the model The trade-off condition for the CB from the FOC of the CB minimization problem with the loss function given by (42):

\[ x_t = -\frac{\kappa}{\alpha} \left( \mu \phi + \delta^G \right) \pi_t - \frac{\kappa \mu}{2} \xi \tau + \tilde{x}. \]

Using the Phillips curve (1), and the previous trade-off condition yields:

\[ \pi_t = \frac{\alpha \beta}{\alpha + \kappa^2 (\mu \phi + \delta^G)} E_t^* \pi_{t+1} + \frac{\alpha \kappa}{\alpha + \kappa^2 (\mu \phi + \delta^G)} \tilde{x} - \frac{1}{2} \frac{\kappa^2 \mu}{\alpha + \kappa^2 (\mu \phi + \delta^G)} \xi \tau + \frac{\alpha}{\alpha + \kappa^2 (\mu \phi + \delta^G)} e_t. \]

Substituting the supposed solution of expected inflation, \( E_t^* \pi_{t+1} = E_t \pi_{t+1} = \zeta_0 + \zeta_1 e_t \), into the above equation gives:

\[ \pi_t = \frac{\alpha \beta}{\alpha + \kappa^2 (\mu \phi + \delta^G)} \zeta_0 + \frac{\alpha \kappa}{\alpha + \kappa^2 (\mu \phi + \delta^G)} \tilde{x} - \frac{1}{2} \frac{\kappa^2 \mu}{\alpha + \kappa^2 (\mu \phi + \delta^G)} \xi \tau + \frac{\alpha}{\alpha + \kappa^2 (\mu \phi + \delta^G)} e_t. \]

Comparing this solution with the assumed solution of inflation, i.e., \( \pi_t = \zeta_0 + \zeta_1 e_t \) yields:

\[ \begin{align*}
\zeta_0 &= \frac{\alpha \kappa}{\alpha (1 - \beta) + \kappa^2 (\mu \phi + \delta^G)} \tilde{x} - \frac{1}{2} \frac{\kappa^2 \mu}{\alpha (1 - \beta) + \kappa^2 (\mu \phi + \delta^G)} \xi \tau, \\
\zeta_1 &= \frac{\alpha}{\alpha + \kappa^2 (\mu \phi + \delta^G)}. 
\end{align*} \]

Finally, we obtain the equilibrium solution for inflation under RE:

\[ \pi_t = \frac{\alpha \kappa}{\alpha (1 - \beta) + \kappa^2 (\mu \phi + \delta^G)} \tilde{x} - \frac{1}{2} \frac{\kappa^2 \mu}{\alpha (1 - \beta) + \kappa^2 (\mu \phi + \delta^G)} \xi \tau + \frac{\alpha}{\alpha + \kappa^2 (\mu \phi + \delta^G)} e_t. \]

Using this solution in the targeting rule leads to the solution for the output gap.
under RE:

\[
x_t = \frac{\alpha(1-\beta)}{\alpha(1-\beta)+\kappa^2(\mu\phi+\delta\gamma)} \tilde{x} - \frac{1}{2} \frac{\kappa(1-\beta)}{\alpha(1-\beta)+\kappa^2(\mu\phi+\delta\gamma)} \xi_T - \frac{\kappa(\mu\phi+\delta\gamma)}{\alpha+\kappa^2(\mu\phi+\delta\gamma)} e_t.
\]

Optimal inflation penalty rate under RE  Replacing \(\pi_t\) and \(x_t\) by their respective equilibrium solution into (3) and differentiating the government’s loss function with respect to \(\tau\). We obtain the first-order condition of the government’s minimization problem:

\[
\frac{\partial L_t}{\partial \tau} = \alpha \left[ \left( \frac{\alpha(1-\beta)}{\alpha(1-\beta)+\kappa^2(\mu\phi+\delta\gamma)} \tilde{x} - \frac{1}{2} \frac{\kappa(1-\beta)}{\alpha(1-\beta)+\kappa^2(\mu\phi+\delta\gamma)} \xi_T - \frac{1}{2} \frac{\kappa(1-\beta)}{\alpha+\kappa^2(\mu\phi+\delta\gamma)} \xi_T \right) \right] + \delta^G \left[ \frac{\alpha}{\alpha(1-\beta)+\kappa^2(\mu\phi+\delta\gamma)} \tilde{x} - \frac{1}{2} \frac{\kappa^2}{\alpha(1-\beta)+\kappa^2(\mu\phi+\delta\gamma)} \xi_T \right] = 0.
\]

This implies:

\[
\frac{\partial L_t}{\partial \tau} = 0 \Rightarrow \tau = 2\alpha \kappa \frac{(\mu\phi + \delta\gamma)(1-\beta) - \delta^G}{[\alpha(1-\beta)^2 + \kappa^2 \delta^G] \mu \xi} \tilde{x}.
\]

C.2 The equilibrium solution under learning

The CB minimizes the loss function given in (42) subject to the three constraints, i.e., the Phillips curve (1), IS equation (2) and learning equations (6)-(7). The Lagrangian
of this minimization problem is:

$$ L^P = \beta^t \sum_{i=0}^{+\infty} \left\{ 1 \over 2 \left[ (\mu \phi + \delta^G) \pi_{t+i}^2 - \mu \xi (\tau_0 - \tau \pi_{t+i}) + \alpha (x_{t+i} - \tilde{x})^2 \right] ight. $$

\[ - \lambda_{1,t+i} [\pi_{t+i} - \beta a_{t+i} - \kappa x_{t+i} - e_{t+i}] \]

\[ - \lambda_{2,t+i} \left[ x_{t+i} - b_{t+i} - \sigma^{-1}(r_{t+i} - a_{t+i}), \right] \]

\[ - \lambda_{3,t+i} [a_{t+1+i} - a_{t+i} - \gamma_{t+1+i} (\pi_{t+i} - a_{t+i})] \]

\[ - \lambda_{4,t+i} [b_{t+1+i} - b_{t+i} - \gamma_{t+1+i} (x_{t+i} - b_{t+i})] \]

Using the same method as in A.1, we find the intertemporal trade-off condition for the CB under constant-gain learning as:

$$ \kappa (\mu \phi + \delta^G) \pi_t + \alpha x_t + \frac{1}{2} \kappa \left[ 1 - \beta (1 - \gamma) \right] \mu \xi \tau = \alpha \beta \left[ 1 - \gamma (1 - \beta) \right] E_t x_{t+1} $$

\[ + \kappa \beta (1 - \gamma) (\mu \phi + \delta^G) E_t \pi_{t+1} \]

\[ + \alpha \left[ 1 - \gamma \beta^2 - \beta (1 - \gamma) \right] \tilde{x} \]

We use (A.10) into the previous equation to get rid of expected output gap:

$$ E_t \pi_{t+1} = \frac{\kappa^2 (\mu \phi + \delta^G) + \alpha + \alpha \gamma \beta^2 [1 - \gamma (1 - \beta)]}{\beta \{ \kappa^2 (1 - \gamma) (\mu \phi + \delta^G) + \alpha [1 - \gamma (1 - \beta)] \}} \pi_t $$

\[ - \frac{1}{\beta} \frac{\alpha}{\alpha} \left[ 1 - \beta (1 - \gamma) [1 - \gamma (1 - \beta)] \right] a_t \]

\[ - \frac{1}{\beta} \frac{\alpha}{\alpha} \left[ 1 - \beta (1 - \gamma) [1 - \gamma (1 - \beta)] \right] e_t \]

\[ + \frac{1}{2} \frac{\kappa^2 \mu [1 - \beta (1 - \gamma)]}{\beta \{ \kappa^2 (1 - \gamma) (\mu \phi + \delta^G) + \alpha [1 - \gamma (1 - \beta)] \}} \xi \tau \]

\[ - \frac{1}{\beta} \frac{\alpha}{\alpha} \left[ 1 - \beta (1 - \gamma) [1 - \gamma (1 - \beta)] \right] \tilde{x} \]
We rewrite the expected inflation as

\[ E_t \pi_{t+1} = A_{11} \pi_t + A_{12} a_t + A_{13} \bar{x} + A_{14} \xi_t + P_1 e_t, \]  

(C.1)

where \( A_{11}, A_{12}, A_{13} \) and \( P_1 \) are equal to the coefficients respectively given by (A.13)-(A.16).

The new term \( A_{14} \) is given by:

\[ A_{14} \equiv \frac{1}{2} \frac{\kappa^2 \mu [1 - \beta(1 - \gamma)]}{\beta \{ \kappa^2 (1 - \gamma) (\mu \phi + \delta^2) + \alpha [1 - \gamma (1 - \beta)] \}}. \]  

(C.2)

According to the proposition 1 from Blanchard and Kahn (1980), the solution of the ALM for inflation takes the following form:

\[ \pi_t = c^{cg}_\pi a_t + \Gamma^{cg}_\pi \xi_t + \Omega^{cg}_\pi \bar{x} + d^{cg}_\pi e_t. \]  

(C.3)

We obtain using (6) and (C.3):

\[ E_t \pi_{t+1} = c^{cg}_\pi E_t a_{t+1} + \Omega^{cg}_\pi \bar{x} + \Gamma^{cg}_\pi \xi_t. \]  

(C.4)

Using (C.1) and (C.4) to eliminate \( E_t \pi_{t+1} \) and arranging the terms yield:

\[ \pi_t = \frac{[A_{12,t} - c^{cg}_\pi (1 - \gamma)]}{(c^{cg}_\pi \gamma - A_{11,t})} a_t + \frac{(A_{13} - \Omega^{cg}_\pi)}{(c^{cg}_\pi \gamma - A_{11,t})} \bar{x} + \frac{(A_{14} - \Gamma^{cg}_\pi)}{(c^{cg}_\pi \gamma - A_{11,t})} \xi_t + \frac{P_{1,t}}{(c^{cg}_\pi \gamma - A_{11,t})} e_t. \]  

(C.5)

This implies that: \( c^{cg}_\pi = \frac{A_{12} - c^{cg}_\pi (1 - \gamma)}{c^{cg}_\pi \gamma - A_{11}}, \) \( d^{cg}_\pi = \frac{P_1}{c^{cg}_\pi \gamma - A_{11}}, \) \( \Omega^{cg}_\pi = \frac{A_{13}}{(c^{cg}_\pi \gamma - A_{11} + 1)} \) and

\[ \Gamma^{cg}_\pi = \frac{A_{14}}{(c^{cg}_\pi \gamma - A_{11} + 1)}. \]  

(C.6)

The stability conditions are equivalent to the ones examined in A.1 and A.2.
C.3 The ALM under constant gain learning

The feedback coefficients $c^g_{\pi}, \Omega^g_{\pi}$ and $d^g_{\pi}$ are identical to those in equation (28). We therefore only calculate the one on the inflation penalty rate, $\Gamma^g_{\pi}$.

Substituting $A_{11}$ and $A_{14}$ into (C.6) leads to:

$$\Gamma^g_{\pi} = -\frac{1}{2} \frac{\kappa^2 \mu [1 - \beta(1 - \gamma)]}{\Theta(y)} \tag{C.7}$$

where we define:

$$\Theta(y) = \kappa^2 \left( \mu \phi + \delta^G \right) [1 - \beta(1 - \gamma) - c^g_{\pi} \gamma \beta (1 - \gamma)]$$

$$+ \alpha \{1 - \beta [1 - \gamma (\beta - c^g_{\pi})] [1 - \gamma (1 - \beta)]\}.$$ 

We rewrite the final form of the ALM for inflation with $c^g_{\pi}, d^g_{\pi}$, and $\Omega^g_{\pi}$ already defined by (29), (30) and (31), respectively:

$$\pi_t = c^g_{\pi} a_t + \Omega^g_{\pi} \bar{x} + \Gamma^g_{\pi} \xi \tau + d^g_{\pi} e_t. \tag{C.8}$$

where

$$\Gamma^g_{\pi} = -\frac{1}{2} \frac{\kappa^2 \mu [1 - \beta(1 - \gamma)]}{\Theta(y)}. \tag{C.9}$$

Using the Phillips curve, we replace and obtain the ALM for the output gap:

$$x_t = c^g_{x} a_t + d^g_{x} e_t + \Omega^g_{x} \bar{x} + \Gamma^g_{x} \xi \tau, \tag{C.10}$$

where $c^g_{x}, d^g_{x}, \Omega^g_{x}$ are denoted in the ALM for the output gap (32) and $\Gamma^g_{x} = \Gamma^g_{\pi} \kappa^g$.

Using the New Keynesian IS equation into (C.10), the ALM for the interest rate is:

$$r_t = c^g_{r} a_t + \delta^g_{r} b_t + d^g_{r} e_t + \Omega^g_{r} \bar{x} + \Gamma^g_{x} \xi \tau, \tag{C.11}$$
with $c_{cg}^g$, $d_{cg}^g$, $\delta_{cg}^g$, $\Omega_{cg}^g$ are given in the ALM for the interest rate (33) and $\Gamma_{cg}^g = \frac{-2}{\kappa} \Gamma_{cg}^g$.

**C.4 Properties of the feedback coefficients**

As in the previous appendix with constant gain without the inflation penalty, we still have the following relationship: there is a unique solution for $c_{cg}^g$ so that $0 < c_{cg}^g < \frac{\alpha \beta}{\alpha + \kappa^2 (\mu \phi + \delta G)}$.

**The case where $\gamma = 0$.** Substituting $\gamma = 0$ into (A.13)-(A.16) and using the results in (A.20)-(A.22), we recall that $A_{11} \equiv \frac{1}{\beta}$, $A_{12} \equiv -\frac{\alpha (1-\beta)}{\beta [\kappa^2 (\mu \phi + \delta G) + \alpha]}$, $A_{13} \equiv -\frac{\alpha \kappa (1-\beta)}{\beta [\kappa^2 (\mu \phi + \delta G) + \alpha]}$ and $P_1 \equiv -\frac{\alpha}{\beta [\kappa^2 (\mu \phi + \delta G) + \alpha]}$ and $A_{14}$ is:

$$A_{14} \equiv \frac{1}{2 \beta} \frac{\kappa^2 \mu (1-\beta)}{\kappa^2 (\mu \phi + \delta G) + \alpha},$$

and $c_{cg}^g = \frac{\alpha \beta}{\alpha + \kappa^2 (\mu \phi + \delta G)}$, $d_{cg}^g = \frac{\alpha}{\alpha + \kappa^2 (\mu \phi + \delta G)}$, $\Omega_{cg}^g = \frac{\alpha \kappa}{\alpha + \kappa^2 (\mu \phi + \delta G)}$, $\Gamma_{cg}^g = \frac{1}{2 \alpha + \kappa^2 (\mu \phi + \delta G)}$.

**The case where $\gamma = 1$.** Inserting $\gamma = 1$ into (A.13)-(A.16) and similarly, using the results in (A.20)-(A.22) lead to $A_{11} \equiv \frac{\kappa^2 (\mu \phi + \delta G) + \alpha (1+\beta^3)}{\beta^2 \alpha}$, $A_{12} \equiv -\frac{1}{\beta}$, $A_{13} \equiv -\frac{\alpha \kappa (1-\beta^2)}{\beta^2 \alpha}$, $P_1 \equiv -\frac{1}{\beta^2}$ and:

$$A_{14} \equiv \frac{1}{2 \beta^2 \alpha} \frac{\kappa^2 \mu}{\beta^2 \alpha},$$

and $c_{cg}^g = \frac{\kappa^2 (\mu \phi + \delta G) + \alpha (1+\beta^3) - \sqrt{\kappa^2 (\mu \phi + \delta G) + \alpha (1+\beta^3)^2 - 4 \alpha^2 \beta^3}}{2 \alpha \beta^2}$, $d_{cg}^g = \frac{\alpha}{\kappa^2 (\mu \phi + \delta G) + \alpha [1 + \beta^2 (\beta - c_{cg}^g)]}$.
Ωπ = \frac{\alpha \kappa (1-\beta^2)}{\kappa^2 (\mu \phi + \delta \tau) + \alpha [1-\beta^2 (1+c^g_{\pi}-\beta)]}, \text{and}

\Gamma_{\pi}^{cg} = \frac{-1}{2} \frac{\kappa^2}{\kappa^2 (\mu \phi + \delta \tau) + \alpha [1-\beta^2 (1+c^g_{\pi}-\beta)]}.

C.5 Optimal inflation penalty under learning

Long-term central bank contracting

In this case, the government signs a long-term contract taking expectations as given, with \( E_t \pi_{t+i} = E_t a_t \). Furthermore, the steady state is defined as \( E_t \pi_{t+i} = \pi_t = a_{t+i} = a_t \) and \( e_{t+i} = e_t = 0 \), then the social loss function can be rewritten using these steady-state conditions, the expectation of the ALM for inflation (47), and the fact that \( e^g_x = -\frac{\beta - c^g_{\pi}}{\kappa} \), \( \Gamma^g_x = \frac{\Gamma^g_{\pi}}{\kappa} \) and \( \Omega^g_x = \frac{1}{\kappa} \Omega^g_{\pi} \) from (52). It follows from (C.16):

\[
E_t \pi_{t+i} = \frac{\Gamma^g_{\pi} \xi}{1-c^g_{\pi}} \tau + \frac{\Omega^g_{\pi}}{1-c^g_{\pi}} \tilde{x},
\]

(C.12)

Using \( E_t \pi_{t+i} = E_t a_t \) and (C.12), substituting \( e^g_x = -\frac{\beta - c^g_{\pi}}{\kappa} \), \( \Gamma^g_x = \frac{\Gamma^g_{\pi}}{\kappa} \) and \( \Omega^g_x = \frac{1}{\kappa} \Omega^g_{\pi} \) into equation (C.10) and taking the expectation of the resulting equation in period \( t+i \) leads to

\[
E_t x_{t+i} = \frac{(1-\beta) \xi \Gamma^g_{\pi}}{\kappa (1-c^g_{\pi})} \tau + \frac{(1-\beta) \Omega^g_{\pi}}{\kappa (1-c^g_{\pi})} \tilde{x},
\]

(C.13)

We then introduce in the government’s loss function the expression of expected inflation and the expected output gap:

\[
L^g_t = \frac{1}{2} E_t \sum_{i=0}^{+\infty} \beta^i \left[ \delta^G \pi^2_{t+i} + \alpha (x_{t+i} - \tilde{x})^2 \right]
\]

\[
= \frac{1}{2(1-\beta)} \left[ \delta^G \left( \frac{\Gamma^g_{\pi} \xi}{1-c^g_{\pi}} \tau + \frac{\Omega^g_{\pi}}{1-c^g_{\pi}} \tilde{x} \right)^2 + \alpha \left( \frac{(1-\beta) \xi \Gamma^g_{\pi}}{\kappa (1-c^g_{\pi})} \tau + \frac{(1-\beta) \Omega^g_{\pi}}{\kappa (1-c^g_{\pi})} \tilde{x} - \tilde{x} \right)^2 \right]
\]
The first-order condition of the government’s minimization problem is:

\[
\frac{\partial L^s}{\partial \tau} = 0 \Rightarrow \alpha \left[ \frac{(1 - \beta) \xi \Gamma^g}{\kappa (1 - c^g_\pi)} \tau + \frac{(1 - \beta) \Omega^g_\pi}{\kappa (1 - c^g_\pi)} \tilde{x} - \tilde{x} \right] \left[ \frac{(1 - \beta) \xi \Gamma^g_\pi}{\kappa (1 - c^g_\pi)} \tau + \alpha \left( 1 - \beta \right) \Omega^g_\pi \Gamma^g_\pi \xi \left( 1 - c^g_\pi \right) \right] = 0.
\]

This leads to the optimal inflation penalty rate:

\[
\tau = \frac{\alpha \kappa (1 - \beta) (1 - c^g_\pi) - \left( \frac{\alpha (1 - \beta)^2 + \delta^G \kappa^2}{\alpha (1 - \beta)^2 + \delta^G \kappa^2} \right) \Omega^g_\pi \Gamma^g_\pi \xi}{\alpha (1 - \beta)^2 + \delta^G \kappa^2} \tilde{x} \text{ for } \gamma \in (0, 1).
\]

When \( \gamma = 0 \), \( c^g_\pi = \frac{\alpha \beta}{\alpha + \kappa^2 (\mu \phi + \delta^G)} \), \( \Omega^g_\pi = \frac{\alpha \kappa}{\alpha + \kappa^2 (\mu \phi + \delta^G)} \) and \( \Gamma^g_\pi = -\frac{1}{2} \frac{\kappa^2 \mu}{\alpha + \kappa^2 (\mu \phi + \delta^G)} \), it yields:

\[
\tau = 2 \alpha \kappa \left( \mu \phi + \delta^G \right) (1 - \beta) - \delta^G \left[ \alpha (1 - \beta)^2 + \kappa^2 \delta^G \right] \mu \xi \tilde{x}. \Box
\]

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