Redistributive Effects of Inflation, Output, Unemployment and Labor Force Participation

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October 2017

Abstract

Empirical evidence suggests that inflation has a positive effect on both output and unemployment in the long run in the United States. This paper develops a search-theoretic monetary model with heterogeneous agents in which a higher inflation rate increases both output and unemployment. The model has two key features: (i) separation between workers and employers and (ii) endogenous labor force participation. Changes in money supply redistributes consumption between employers and workers. This redistribution along with endogenous labor force participation creates a channel by which a higher inflation rate increases output, unemployment, and labor force participation. Though output rises, inflation acts as a regressive consumption tax. Consumption and welfare of workers fall. The Friedman rule does not maximize social welfare.

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1 Introduction

There is a large empirical literature which has examined the long-run relationship between inflation and real activities. Empirical evidence for the United States suggests a positive long run relationship between inflation and output (Ahmed and Rogers 2000, Ericsson et. al. 2001, Bashar 2011).[1] At the same time, there are parallel studies which have examined relationship between inflation and unemployment. These studies find a positive relationship between inflation and unemployment in the United States (Friedman 1977, Beyer and Farmer 2007, Berentsen, Menzio, and Wright 2011, Huang and King 2014). Taken together, these studies suggest that a higher inflation rate increases both output and unemployment.

There is also a large theoretical literature which has studied the long-run relationship between inflation and real activities (superneutrality). The predicted relationship between inflation and real activities depends on underlying monetary framework. Cash-in-Advance models usually predict a negative relationship between inflation and real activities. A higher inflation rate reduces output and employment (e.g. Stockman 1981, Cooley and Hansen 1989). In the Money-in-Utility function models, money is superneutral (Sidrauski 1967). Search theoretic monetary models which incorporate labor markets (e.g. Kumar 2008, Berentsen, Menzio, and Wright 2011) typically predict a negative relationship between inflation and output and a positive relationship between inflation and unemployment. The divergent effects of inflation on output and unemployment in these models are at odds with empirical evidence.

In this paper, I develop a search-theoretic monetary model with heterogeneous agents in which changes in inflation rate have positive effects on both output and unemployment.[2] There are two key features of the model. Firstly, only a fraction of agents, called employers, own firms. Firms require labor to produce goods. Labor is supplied by other type of agents (non-owners)

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[1]Evidence from other developed countries also suggests a long run positive relationship between inflation and output (Bullard and Keating 1995, Ahmed and Rogers 2000, Ericsson et. al. 2001, Rapach 2003, Bashar 2011).

[2]Search theory is currently the dominant paradigm for the micro-foundation of money. Search-theoretic models explicitly model the patterns of meetings, specialization in production and consumption, and the information structure which lead to the 'double coincidence of wants problem' in the goods market, and intrinsically useless (fiat) money emerges as a medium of exchange endogenously (see Kiyotaki and Wright 1993, Shi 1997, Lagos and Wright 2005).

called workers. Secondly, the labor force participation is endogenous.

The separation between workers and owners of firms is similar to Diamond and Yellin (1990), Laing, Li, and Wang (2007), and Ghossoub and Reed (2017) and is quite realistic. There is substantial empirical evidence that only a minority of households participate in capital markets. Mankiw and Zeldes (1991) using PSID data find that only one fourth of households owned stocks directly or indirectly. Brav, Constantinides, and Geczy (2002) using Consumer Expenditure Survey data from 1980-2000 find that only a small section (15 − 20%) of the U.S. households owned financial assets such as stocks, bonds, mutual funds, and other securities. Of those who held such financial assets, only 10-15 percent of them held assets worth more than $2000 (in 1996 dollars). Mulligan and Sala-i-Martin (2000) argue that for the majority of households in the United States, the relevant monetary decision is not what fraction of assets should be held in interest bearing assets, but whether to hold any such asset at all. Attansaio et. al. (2002) find similar evidence for the United Kingdom.

Empirical evidence shows that changes in the labor force participation have been important drivers of both long-term and cyclical movements in output and unemployment in the United States (Shapiro and Watson 1988, Foroni et. al. 2015). Endogenizing labor force participation is crucial to understand the effects of monetary policy on real activities. In my model, separation between workers and employers households and endogenous labor force participation induce novel effects of changes in money creation rate on output, unemployment, and labor force participation. A higher money creation rate leads to higher output, unemployment, and labor force participation.

In the model there are two markets: goods market and labor market. Both markets are characterized by search frictions. The matching process and price determination in both markets are modeled along the lines of competitive equilibrium analyzed in Lucas and Prescott (1974), Alvarez and Veracierto (1999), Rocheateau and Wright (2005), and Kambourov and Manovskii (2009). Agents queue to enter markets. But due to frictions only a fraction of them are able to enter these markets. Agents in both markets are assumed to be price takers. Money circulates as a medium of exchange in both the goods and labor markets. Money is supplied by the government and it distributes newly created money among workers and employer households in lump-sum fashion. Worker and employer households may receive differential transfers.

In the paper, I derive following main results. Firstly, an increase in the money creation rate increases output, labor force participation rate, and unemployment rate. These results are consistent with the empirical evidence.[3] Secondly, inflation acts as a regressive consumption tax. A higher money creation rate redistributes consumption in favor of employers increasing their welfare. Consumption and welfare of workers fall despite higher output. The results that inflation increases inequality and its cost is mainly born by non-asset holders are supported by empirical evidence (Romer and Romer 1999, Easterly and Fischer 2001, Albanesi 2007, Ghossoub and Reed 2017). Finally, the Friedman rule which requires that the money creation rate be equal to the discount rate does not maximize social welfare.

The mechanism of these results is as follows. Firstly, a higher money creation increases money holding of employers for a *given output*. Thus, they need to use less of their sales proceed to finance their wage-bill. It reduces consumption of workers, which induces workers to supply more labor by increasing their marginal utility of consumption (*the income effect of inflation on labor supply*). Consequently, the labor force participation and labor hours worked per employed worker rise. But at the same time a higher money creation rate erodes the value of real wages, which induces workers to supply less labor reducing labor force participation and labor hours worked per employed worker (*the substitution effect of inflation on labor supply*). The net effect of a higher money creation rate depends on the relative strength of these two effects.

I find that the income effect dominates the substitution effect and a higher money creation rate leads to higher output, labor force participation, and labor hours worked per employed worker and lower consumption and welfare of worker households. At the same time, a higher labor force participation increases the number of workers searching for jobs leading to higher unemployment rate. Endogenous labor force participation plays a crucial role in generating positive association between output and unemployment in the model. With *fixed* labor force participation, the model will generate negative association between the two.

Regarding the optimality of Friedman rule, for a *given* distribution of consumption and labor force participation rate, the Friedman rule induces

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[3]Section 7 provides empirical evidence on the long run relationship between inflation, output, employment, unemployment rate, and labor force participation rate in the United States.

optimal employment. But it does not lead to the optimal distribution of consumption and labor force participation.

The result that a higher money creation rate may increase output is also derived in a monetary search-theoretic framework in Shi (1998). However, in his model higher output is accompanied by lower unemployment. In a very different model, Laing et. al. (2007) also find that a higher money creation rate may increase output. In their model, households care about consumption variety and money increases the variety of goods consumed. In their model there is no unemployment. Ghossoub and Reed (2017) incorporate Mundell-Tobin effect in a monetary model and they find that higher inflation leads to higher output. Interestingly, in their model a higher inflation also increases consumption inequality between workers and employers. In their model, labor supply is exogenous and their is no unemployment.

In terms of mechanism, in both Shi (1998) and Laing et. al. (2007) the positive relationship between inflation and output arises due to complementarity between household labor supply and its search intensity in the goods market. Higher inflation can encourage both labor supply and search effort resulting in a higher output. Unlike these models, I assume that the search-intensity of buyers in the goods market is *fixed*. The positive relationship between inflation and output arises due to redistribution of consumption between workers and employers rather than the complementarity between labor supply and search effort in the goods market. In Ghossoub and Reed (2017) higher inflation induces agents to rebalance their savings towards capital away from real money balances.[4] My results do not rely on substitution between money and capital.

My model relates to monetary search models such as Kumar (2008) and Berentsen, Menzio, and Wright (2011), which incorporate unemployment in monetary search framework. In these models, there is no separation between employers and workers. An agent supplies labor as well as own firms. This precludes inflation from having any redistributive effect. In addition, labor force participation in these models is exogenous. In terms of result, as mentioned earlier, these models predict a negative relationship between inflation and output, which is at odds with empirical evidence.

My results relate to a number of studies (Erosa and Ventura 2002, Al-

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[4]There are number of studies which incorporate in Mundell-Tobin effect in monetary model (e.g. Weiss 1980, Espinosa-Vega and Russell 1998, Heer 2003). However, Weiss (1980) and Espinosa-Vega and Russel (1998) do not have unemployment. In Heer (2003), higher output leads to lower unemployment.

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banesi (2007), Boel and Camera 2009, see also Ghossoub and Reed 2017), which find that inflation acts as regressive consumption tax. However, in Erosa and Ventura (2002) and Boel and Camera (2009) labor supply is exogenous and there is no unemployment. Albanesi (2007) builds a political economy model in which policies are set by bargaining between low-income and high income households. She shows that conflict over policies results in high inflation and high inequality.

My results are also part of a literature which studies the environments with heterogeneous agents in which the Friedman rule is sub-optimal (e.g. Bhattacharya, Haslag, Martin 2005, Green and Zhou 2005, Molico 2006). Similar to these studies, I find the Friedman rule is not optimal, though in a very different environment. These studies examine the optimality of the Friedman rule in endowment economies.

Rest of the paper is organized as follows. Section 2 describes the model. Section 3 analyzes the optimal decisions of agents. Section 4 characterizes and establishes the existence of a symmetric and stationary steady state monetary equilibrium. Section 5 analyzes the effects of changes in money creation rate on output, unemployment and labor force participation. Section 6 analyzes effects of money creation rate on distribution and welfare. Section 7 provides empirical evidence on the long run relationship between inflation, output, unemployment rate, and labor force participation rate in the United States. This is followed by concluding remarks. All proofs are in appendix.

2   The Economy

Time is discrete. Consider an economy with $K(\geq 2)$ non-storable goods consisting of two islands: H and F. Island H is inhabited by a large number (unit measure) of infinitely-lived risk-averse agents called workers. Workers desire to consume goods and supply labor. However, they cannot produce goods. Workers differ in terms of goods they desire to consume. The type of good a worker would like to consume is determined by a uniformly distributed *i.i.d.* random shock each period. Thus in any period, a particular type of good is consumed by $\frac{1}{K}$ workers.

Island F consists of a large number (unit measure) of K types of infinitely-lived firms. Firms possess technology to produce consumption goods but require labor supplied by workers to do so. A firm of particular type produces goods of particular type. In any period, a particular type of good is produced

by $\frac{1}{K}$ firms. Similar to Diamond and Yellin (1990) and Laing et. al. (2007), I assume that each firm is owned by infinitely-lived agents called employers (owners) and employers desire to consume the product produced by his own firm. Employers live on island F. Assume that claims on profits of firms (or shares) cannot be used to buy goods in the goods market.[5]

There are two markets: the goods market and the labor market. Both markets are characterized by search frictions. Buyers and sellers in the goods market and workers and employers in the labor market are brought together randomly through the search process described later. Random meetings between buyers and sellers imply that a particular buyer or a seller in the goods market cannot be relocated in future. This assumption rules out credit arrangements between buyers and sellers and exchanges must be *quid-pro-quo*. Thus, money is used as a medium of exchange in both goods and labor markets.

The assumption that employers consume their own product is equivalent to assuming that employers consume goods which are different from what they produce, but they can trade in a centralized goods market without money (cash). However, worker households can buy goods only in the decentralized goods market, where money is needed. These assumptions are similar to Erosa and Ventura (2002), Albansi (2007), and Boela and Camera (2009) and allow us to study the distributional consequences of inflation. These assumptions are consistent with considerable empirical evidence that a significant fraction of families in the US do not use checking account or credit cards to purchase goods and majority of them were low-income income families. Avery et. al. (1987) using data from the 1986 Survey of Currency and Transaction Accounts Usage report that the majority of purchases by households (82%) in the US were done using M1 and 46% of households did not use credit card. Mulligan and Sala-i-Martin (2000) using the Survey of Consumer Finance, 1989 find that 25% of households in the US did not have checking account with a positive balance. Kennickell et. al. (1997) and Aizcorbe et. al. (2003) report that around 10% of US families did not have any type of bank account.

Due to random meetings, individual agents in both goods market and labor market face uncertainty in trading outcomes. This generates non-

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[5]This restriction is needed so that these claims do not replace money as a medium of exchange. One can assume that worker households can easily counterfeit such claims and sellers in the goods market cannot verify them (e.g. Aruba et. al. 2008).

degenerate distributions of money holding, sales, employment status, and consumption which make the model analytically intractable and numerically challenging. In order to make these distributions degenerate and the analysis tractable, following Shi (1997, 1998), I use the construct of large worker households and firms.[6]

Each worker household is assumed to comprise of unit measure of workers and unit measure of buyers. These workers and buyers do not have independent preferences. Rather, the household prescribes their trading strategies to maximize the overall household utility. Workers and buyers share equally in the utility generated by the household consumption. With this modeling device, decisions of different worker household types are identical in a symmetric equilibrium, except for the types of goods they consume. Thus, I can analyze the behavior of a representative worker household.

Similarly, a firm consists of unit measure of sellers who sell goods in the goods market. Just as in the case of worker households, these agents do not have independent preferences, but undertake activities in order to maximize firm's profit.[7] Large number of sellers implies that idiosyncratic risks faced by individual sellers are smoothed within the firm. With this construction of firms, decisions of employers of different types are identical in a symmetric equilibrium, except for the types of goods they produce and consume. Thus, I can analyze the behavior of a representative employer (or firm).

2.1 Trading and Price Determination in the Goods and Labor Markets

I model the pricing mechanism in both markets along the lines of the competitive equilibrium analyzed in Lucas and Prescott (1974), Alvarez and Veracierto (1999), Rocheteau and Wright (2005), and Kambourov and Manovskii (2009). In this formulation, markets are assumed to be competitive. However, due to search frictions only a fraction of agents are able to enter market. Agents are assumed to be price takers. Price is determined by the condition that total demand equals total supply.

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[6]An alternative frameworks which produces degenerate distributions of money holding and prices is examined by Lagos and Wright (2005).

[7]These sellers need not be unpaid. One can assume that a *fixed number* of workers are required for sales activities. These employees are chosen randomly at the beginning of every period from the existing pool of employees.

The search process in the goods market is modeled as in Rocheteau and Wright (2005). Similar to them, I assume that buyers and sellers in the goods market queue to enter the market every period and only a fraction of them are able to enter the market. Buyers and sellers who are able to enter the market trade at the given price, $\hat{p}_t(i)$, for good i. Suppose that only a *fixed* fraction ξ ($0 < \xi < 1$) of buyers and sellers are able to enter the market.

The assumption that only a *fixed* fraction of buyers and sellers enter the market allows me to clearly contrast my results and mechanism from the studies (*e.g.* Shi 1998, Laing et. al. 2007), which focus on the complementarity between the labor supply decision and the search-intensity of households in the goods market. This issue is further discussed in the concluding section. Since, measures of buyers in a worker household and sellers in a firm are normalized to one, the measures of buyers in a worker household and sellers of a firm who are able to trade are equal to ξ in any time period.

I model search process in the labor market along the lines of Lucas and Prescott (1974), Alvarez and Veracierto (1999), and Kambourov and Manovskii (2009). Workers queue to enter the labor market every period and only a fraction of them, χ, are able to enter the market. Workers who are able to enter the market are randomly allocated among employers. Suppose that χ is a following function of aggregate number of workers queuing for employment, $\hat{n}_t$, i.e.

$$\chi(\hat{n}_t) = a\hat{n}_t^{-\zeta}, \text{ where } a > 0 \ \& \ 0 < \zeta < 1. \tag{2.1}$$

I call $\chi(\hat{n}_t)$ the entry function in the labor market.[8] Denote average price in goods market by $\hat{p}_t$ and real wage per unit of labor in the labor market by $\hat{w}_t$ at time t. Since, I am going to focus on symmetric equilibrium in which $\hat{p}_t(i) = \hat{p}_t$, I will drop the goods index i from prices of individual goods in the rest of the paper. In addition, I will denote the variables, which are taken as given by a particular worker household or an employer, with superscript "^".

2.2 Money Supply Process

Let $\hat{M}_t$ be the money supply at time t. Suppose that the government increases money supply at the constant rate g and thus $\hat{M}_{t+1} = g\hat{M}_t$. Worker

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[8]The scaling factor 'a' is used to ensure that $\chi(\hat{n}_t)$ lies between 0 and 1.

households and employers receive monetary injection in lump-sum fashion. These two types of agents may receive differential transfers.

Suppose that at the initial period, an employer has an endowment of $\phi \hat{M}_0$ amount of money, where $0 \leq \phi \leq 1$. A worker household has an initial endowment of money equal to $(1 - \phi)\hat{M}_0$. At the beginning of each subsequent period, an employer receives $(g - 1)\phi \hat{M}_{t-1}$ units of fiat money from the government as a lump-sum transfer. Similarly, a worker household receives $(g - 1)(1 - \phi)\hat{M}_{t-1}$ units of fiat money from the government as a lump-sum transfer.

This formulation encompasses different possibilities. If $\phi = \frac{1}{2}$, both worker households and employers receive identical transfer as in standard monetary models. If $\phi = 1$, then employers/firms receive all the monetary injection as in limited participation models (e.g. Fuerst 1992, Christiano, Eichenbaum, and Evans 1997).

In the paper, I show that the effects of changes in money supply depends on ϕ and changes in ϕ itself has important implications. One way to thinks about ϕ is that the government transfers newly created money to financial institutions (not modeled here), and inhabitants of two islands have differential access to these institutions. Parameter ϕ captures the differential access of inhabitants of these two islands to financial institutions. The other way to think about is that the government provides lump-sum subsidy to workers and employers which is financed by newly created money. The government may provide differential subsidy to workers and employers.

For future use, I call the parameter ϕ distributional parameter of money supply. Note that when the money creation rate, $g < 1$ (decrease in money supply), the distributional parameter, ϕ, determines the proportion in which withdrawal of money is distributed between worker households and employers.

3   Optimal Decisions of the Representative Worker Household and the Representative Employer

3.1   Timing

At the beginning of period, t, both the representative worker household and the representative employer receive monetary transfers. Let M_t^h and M_t^f be units of post-transfer money with the representative worker household and

the representative employer respectively at time t. After receiving transfers, the worker household chooses labor force participation rate (number of workers to participate in the labor market), n_t, and labor intensity (labor units/hours to be supplied) of an employed worker, l_t, at a given real wage $\hat{w}_t$. After receiving instructions, workers go to the labor market and fraction $\chi(\hat{n}_t)$ of workers get employment. Each employed worker then supplies l_t units of labor. Assume that once the workers go to the labor market, they remain separated from the household till the end of period t.

After workers have left the household, the worker household receives preference shock which determines the good which the household would like to consume in period t. After the preference shock, the worker household distributes available money balance, M_t^h, equally among buyers and chooses the amount of money to be spent by a buyer, m_t. After receiving instructions, buyers go to the goods market. Buyers who are able to enter the goods market (fraction ξ) buy goods at the given price, $\hat{p}_t$. For future reference, I call buyers who are able to enter the goods market '*matched buyers*'.

Similarly after receiving transfers, the employer chooses the number of labor units to hire, e_t, taking wages as given and produces output, $f(e_t)$. After production, the employer chooses quantity, q_t, to sell. He distributes the chosen quantity, q_t, equally among sellers. Since the measure of sellers is unity, each seller receives, q_t, units of goods. After receiving goods, sellers go to the goods market. Sellers who are able to enter the goods market (fraction ξ) sell goods at the given price, $\hat{p}_t$. I call sellers who are able to enter the goods market '*matched sellers*'. Since only ξ fraction of sellers are able to sell their goods, the actual sales by a firm is ξq_t. To contrast the actual amount of goods sold from the optimally chosen quantity of goods to sell, q_t, I call q_t the *desired quantity of goods to sell*.

After trading in the goods market, buyers come back to the household with the purchased goods and any residual nominal money balances. Similarly, sellers come back with their nominal sales receipts and any unsold stock of goods. The employer pays wages to employed workers in terms of money and they return to their households.

The residual nominal money balances of buyers and wage receipts of workers are added to the worker household nominal money balance for the next period. Similarly, nominal sales receipts of sellers *net of* nominal wage payment are added to the nominal money balance of the employer to be carried to the next period. Consumption takes place. Time moves to the next period.

3.2 The Optimal Decisions of the Representative Worker Household

Assume that the representative worker household maximizes the discounted sum of utilities from the sequence of consumption less the disutility incurred from working. The household's inter-temporal utility is represented by

$$\sum_{t=0}^{\infty} \left(\frac{1}{1+r} \right)^t \left[U(c_t^h) - \chi(\hat{n}_t) n_t \mu(l_t) - \tau n_t \right] \text{ with } U'(c_t^h) > 0$$

$$U''(c_t^h) < 0, \ \lim_{c_t^h \to 0} U'(c_t^h) > 0,$$

where r, $U(c_t^h)$, $\mu(l_t)$, and τ are the rate of time-preference, the utility derived from consumption, c_t^h, the disutility from supplying l_t units of labor by an employed worker and the disutility from labor force participation respectively.

Let the disutility from supplying l_t units of labor by an employed worker be the following function

$$\mu(l_t) = l_t^\theta \text{ with } \ \theta > 1. \tag{3.1}$$

The money spent by an individual buyer in the goods market satisfies following inequality

$$m_t \leq M_t^h. \tag{3.2}$$

Recall that the measure of matched buyers in the household is ξ. Then consumption, c_t^h, satisfies following inequality

$$c_t^h \leq \xi \frac{m_t}{\hat{p}_t}. \tag{3.3}$$

The budget constraint of the worker household is given by

$$M_{t+1}^h \leq M_t^h + (g-1)(1-\phi)\hat{M}_t + \chi(\hat{n}_t)\hat{p}_t\hat{w}_t n_t l_t - \xi m_t. \tag{3.4}$$

The term on the left hand side is the post-transfer money holding at the beginning of period, $t+1$. The first term on the right hand side is the nominal money balance of the household at time t, the second term is the lump-sum monetary transfer at the beginning of period $t+1$, and the third term is the nominal wage payment received by the household. The final term is the money spent by the matched buyers at time t.

Next I set up the optimization problem of the worker household. Taking real wage, $\hat{w}_t$, prices in the goods market, $\hat{p}_t$, and the initial money holding, $(1-\phi)\hat{M}_0$, as given, the representative worker household chooses the sequence of $\{c_t,\ m_t,\ l_t,\ n_t,\ M_{t+1}^h\}$, $\forall\ t \geq 0$ to solve the following problem.

Worker Household Problem (PH)

$$\max_{c_t^h, m_t, l_t, n_t, M_{t+1}^h} \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t \left[U(c_t^h) - \chi(\hat{n}_t)n_t\mu(l_t) - \tau n_t\right]$$

subject to the constraints on money spent by an individual buyer (3.2), the household's consumption (3.3), and the budget constraint (3.4).

Turning to the optimal choices, consumption, c_t^h, is given by the equality constraint (3.3). Denote the Langrangian multipliers associated with the constraints on the nominal money balance of an individual buyer (3.2) by λ_t and on the household budget constraint (3.4) by ω_{Mt}^h. Then the first order condition for the optimal choice of M_{t+1}^h is given by

$$\omega_{Mt}^h = \frac{1}{1+r}\left[\omega_{Mt+1}^h + \xi\lambda_{t+1}\right]. \tag{3.5}$$

The first order condition has the usual interpretation. The right hand side of (3.5) is the discounted expected marginal benefit from carrying an additional unit of money next period. If the household carries one additional unit of money next period, then it relaxes the budget constraint (3.4) as well as the constraint on the nominal balance of matched buyers (3.2). Note that only ξ fraction of buyers are able to enter the goods market.

The optimal choice of spending by an individual buyer, m_t, satisfies

$$\lambda_t = \frac{U'(c_t^h)}{\hat{p}_t} - \omega_{Mt}^h. \tag{3.6}$$

λ_t can be interpreted as the net surplus generated by a matched buyer for the household from an additional unit of expenditure. Spending of an additional unit of money increases the household's utility by $\frac{U'(c_t^h)}{\hat{p}_t}$, but at the same time it tightens the budget constraint. For a matched buyer to get positive surplus *i.e.*, $\lambda_t > 0$, the constraint on the spending of a matched buyer given in (3.2) must be binding.

The optimal choice of labor intensity, l_t, satisfies following first order condition:

$$\mu'(l_t) = \hat{p}_t \hat{w}_t \omega_{Mt}^h. \tag{3.7}$$

The left hand side of (3.7) is the marginal disutility from working. The right hand side is the marginal benefit. The household receives $\hat{p}_t \hat{w}_t$ from an additional unit of labor supply, which relaxes matched buyers expenditure constraint (3.2) and the household's budget constraint (3.4).

Finally, the optimal choice of labor force participation, n_t, satisfies following first order condition

$$\chi(\hat{n}_t)\mu(l_t) + \tau = \chi(\hat{n}_t)\hat{p}_t \hat{w}_t l_t \omega_{Mt}^h. \tag{3.8}$$

The left hand side of (3.8) is the marginal disutility from labor force participation which takes into account labor force participation cost as well as expected disutility from working. The right hand side is the expected marginal cost, which takes into account that a labor force participant finds employment with probability, $\chi(\hat{n}_t)$.

3.3 The Optimal Decisions of the Representative Employer

Assume that the representative employer maximizes the discounted sum of utilities from the sequence of consumption, c_t^f:

$$\sum_{t=0}^{\infty} \left(\frac{1}{1+r} \right)^t V(c_t^f) \text{ where } V'(c_t^f) > 0, \ V''(c_t^f) \leq 0.$$

Suppose that the representative employer has a production function given by, $f(e_t)$, with following properties

$$f(0) = 0, \ f'(e_t) > 0, \ f''(e_t) < 0, \ \lim_{e_t \to 0} f'(e_t) = \infty, \ \lim_{e_t \to 0} e_t f'(e_t) < \infty \tag{3.9}$$

where e_t is the number of labor units employed at time t. Number of labor units employed is given by the product of number of employed workers and the average number of labor units supplied by them.

Given that only ξ fraction of sellers are able to enter the goods market, consumption by the employer, c_t^f, satisfies following equality:

$$c_t^f = f(e_t) - \xi q_t. \tag{3.10}$$

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Recall that the employer can use the current sales receipt and current post transfer money holding to finance his wage bill. Thus, the employer faces following wage finance constraint:

$$\hat{p}_t \hat{w}_t e_t \leq M_t^f. \tag{3.11}$$

Finally the budget constraint of the employer is given by

$$M_{t+1}^f \leq M_t^f + (g-1)\phi \hat{M}_t + \xi \hat{p}_t q_t - \hat{p}_t \hat{w}_t e_t. \tag{3.12}$$

The term on the left hand side is the post-transfer money holding at the beginning of period, $t + 1$. The first term on the right hand side is the nominal money balance of the employer at time t, the second term is the lump-sum monetary transfer at the beginning of period $t + 1$, and the third term nominal sales receipt. The final term is the nominal wage paid by the employer.

Next I set up the optimization problem of the employer. Taking the real wage, $\hat{w}_t$, the prices in the goods market, $\hat{p}_t$, and the initial money holding, $\phi \hat{M}_0$, as given, the employer chooses the sequence of $\{c_t^f, q_t, M_{t+1}^f, e_t\}$, $\forall\, t \geq 0$ to solve the following problem.

Employer's Problem (PE)

$$\max_{c_t^f, e_t, q_t, M_{t+1}^f} \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t V(c_t^f)$$

subject to the constraints on the employer's consumption (3.10), wage finance constrain (3.11), and the budget constraint (3.12).

Turning to the optimal choices, consumption, c_t^f, is given by the equality constraint (3.10). Denote the Langrangian multipliers associated with the constraints on the wage finance (3.11) by Ω_{wt} and on the employer's budget constraint (3.12) by ω_{Mt}^f. Then the first order conditions for the optimal choices of e_t, q_t, and M_{t+1}^f are given by

$$e_t: \; V'(c_t^f)f'(e_t) = \hat{p}_t \hat{w}_t \Omega_{wt} + \hat{p}_t \hat{w}_t \omega_{Mt}^f; \tag{3.13}$$

$$q_t: \; V'(c_t^f) = \hat{p}_t \omega_{Mt}^f; \tag{3.14}$$

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$$M_{t+1}^f : \omega_{Mt}^f = \frac{1}{1+r}\left[\omega_{Mt+1}^f + \Omega_{wt+1}\right]. \tag{3.15}$$

(3.13) equates the marginal benefit of hiring an additional unit of labor, $V'(c_t^f)f'(e_t)$, with its marginal cost. The employer has to pay nominal wage, $\hat{p}_t\hat{w}_t$, which tightens the constraint on the wage finance (3.11) and the budget constraint (3.12). Ω_{wt} can be interpreted as the net surplus to the employer from hiring.

(3.14) similarly equates the marginal cost of selling an additional unit of good with the marginal benefit. The utility cost of selling an additional unit is $V'(c_t^f)$. The selling of an additional unit increases the money holding by $\hat{p}_t$, which relaxes the budget constraint, the value of which is $\hat{p}_t\omega_{Mt}^f$. (3.15) can be interpreted similarly. An additional unit of money carried forward relaxes the next period wage finance constraint as well as the budget constraint.

3.4 Price and Wage Determination

Since the goods market clears, the demand for goods should be equal to the supply. Recall that only fraction ξ of buyers and sellers are able to enter the goods market. Thus, the total demand for goods is $\frac{\xi\hat{m}_t}{\hat{p}_t}$ and the total supply is $\xi\hat{q}_t$. Price, $\hat{p}_t$, is given by

$$\hat{p}_t = \frac{\hat{m}_t}{\hat{q}_t}. \tag{3.16}$$

Similarly, real wage, $\hat{w}_t$, is determined by the labor market clearing condition

$$\hat{e}_t = \chi(\hat{n}_t)\hat{n}_t\hat{l}_t. \tag{3.17}$$

The left hand side of (3.17) is the total demand of labor units. The right hand side is the total supply of labor units. Due to friction, only fraction $\chi(\hat{n}_t)$ of the workers are able to enter the labor market. $\hat{l}_t$ is the average number of labor units supplied by the workers who are able to enter the labor market.

4 Symmetric Stationary Monetary Equilibrium

This paper restricts its attention to a symmetric and stationary monetary equilibrium. First, I require that all worker households have a common marginal value of money, ω_{Mt}^h, consumption, c_t^h, labor force participation, n_t, and labor intensity, l_t. Similarly, all employers have a common marginal value of money, ω_{Mt}^f, consumption, c_t^f, and employ same number of labor units, e_t. Finally, money has value *i.e.*, the marginal value of real money balance to both worker households and employers be strictly positive, $\hat{p}_t\omega_{Mt}^h, \hat{p}_t\omega_{Mt}^f > 0$.

Denote the supply of real money balance, $\hat{M} \equiv \frac{\hat{M}_t}{\hat{p}_t}$, the real money balance with a worker household, $M^h \equiv \frac{M_t^h}{\hat{p}_t}$, the real money balance with an employer, $M^f \equiv \frac{M_t^f}{\hat{p}_t}$, the real money balance with a buyer, $m = \frac{m_t}{\hat{p}_t}$, the buyer's surplus per purchase, $\hat{p}_t\lambda_t = \lambda$, the marginal value of real money balance for a worker household, $\Omega_M^h \equiv \hat{p}_t\omega_{Mt}^h$, and the marginal value of real money balance for an employer, $\Omega_M^f \equiv \hat{p}_t\omega_{Mt}^f$. From now on, I drop the subscript t from real variables.

For a symmetric stationary monetary equilibrium to exist, the surplus to a matched buyer must be positive (*i.e.*, $\lambda > 0$). If the matched buyer does not receive a positive surplus ($\lambda = 0$), then (3.5) implies that either the marginal value of real money balance to the worker household, $p_t\omega_{Mt}^h = 0$ or $p_t\omega_{Mt}^h$ grows at the rate $(1+r)g$. Similarly, an employer must get strictly positive surplus from hiring (*i.e.*, $\Omega_w > 0$). If $\Omega_w = 0$, then (3.15) implies that either $p_t\omega_{Mt}^f = 0$ or $p_t\omega_{Mt}^f$ grows at the rate, $(1+r)g$. Thus in a symmetric stationary monetary equilibrium, buyers' nominal cash balance expenditure constraint (3.2) and employer's wage financing constraint (3.11) will be binding.

Using (3.5) and (3.6) one can easily show that for $\lambda > 0$, the money creation rate, g, should be greater than $\frac{1}{1+r}$. In the rest of the paper, I impose this condition.

Assumption 1: The money creation rate, $g > \frac{1}{1+r}$.

Assumption 1 also ensures that $\Omega_w > 0$ (see equation 4.6 below). Given buyers' nominal cash balance expenditure constraint (3.2) and the goods market clearing constraint (3.16), the price level is given by $\hat{p}_t \equiv \frac{\hat{M}_t^h}{\hat{q}}, \forall\, t$, in

the stationary and symmetric equilibrium. Also the average price level, $\hat{p}_t$, will grow at the rate equal to the money creation rate *i.e.*, the inflation rate

$$\frac{\hat{p}_{t+1}}{\hat{p}_t} = g \quad \forall t. \tag{4.1}$$

Definition: A symmetric stationary monetary equilibrium (SSME) is defined as a collection of the worker household's choice variables $X^h \equiv \{c^h, M^h, m, l, n\}$, the employer's choice variables, $X^f \equiv \{c^f, M^f, q, e\}$, the prices in the goods market, $\hat{p}_t$, and the real wage in the labor market, $\hat{w}$, and the aggregate variables $\hat{X}^h$ and $\hat{X}^f$, such that

- Given aggregate variables, $\hat{X}^h$ and $\hat{X}^f$, and price in the goods market, $\hat{p}_t$, and real wage in the labor market, $\hat{w}$, the worker household's choice variables X^h solve (PH);

- Given aggregate variables, $\hat{X}^h$ and $\hat{X}^f$, price in the goods market, $\hat{p}_t$, and real wage in the labor market, $\hat{w}$, the employer's choice variables X^f solve (PE);

- the price in the goods market, $\hat{p}_t$, satisfies the goods market clearing condition (3.16);

- the real wage, $\hat{w}$, satisfies the labor market clearing condition (3.17);

- aggregate variables are equal to the relevant worker household's and employer's variables, $\hat{X}^h = X^h$, $\hat{X}^f = X^f$; and

- the marginal values of real money balances to the worker household and the employer, Ω^h_M & Ω^f_M, be strictly positive and finite, $0 < \Omega^h_M, \Omega^f_M < \infty$.

Given that both goods and labor markets clear, money market also clears and thus $\hat{M} = \hat{M}^h + \hat{M}^f$. From now on I suppress "ˆ" from the aggregate variables.

(3.5) and (3.6) imply that the marginal value of real money balance to the worker household, Ω^h_M, is given by

$$\Omega_M^h = \frac{\xi}{(1+r)g - 1 + \xi} U'(c^h). \tag{4.2}$$

(3.7) and (4.2) imply that labor intensity, l, satisfies

$$\mu'(l) = \frac{\xi}{(1+r)g - 1 + \xi} wU'(c^h). \tag{4.3}$$

(4.3) is one of the key equations of the model. It shows that for a given real wage, w, and consumption of the worker household, c^h, a higher money creation rate, g, reduces labor intensity. Intuitively, a higher money creation rate, g, reduces the value of real money balance and thus the worker household reduces labor intensity. For the similar reason, a lower real wage reduces labor supply for a given level of money creation rate, g, and worker household consumption, c^h. Finally, for a given level of real wage, w, and money creation rate, g, a lower consumption level of the worker household, c^h, increases labor intensity. A lower consumption level increases the marginal utility of consumption and thus the marginal return from working.

Given that $\mu(l) = l^\theta$ and $\chi(n) = an^{-\varsigma}$, (3.7) and (3.8) imply that the labor force participation rate, n, satisfies

$$1 + \frac{\tau}{al^\theta n^{-\varsigma}} = \frac{\mu'(l)l}{\mu(l)} \equiv \theta. \tag{4.4}$$

(3.3) and the goods market clearing condition (3.16) imply that consumption of the worker household, c^h, is given by

$$c^h = \xi q. \tag{4.5}$$

From (3.14) and (3.15) I have

$$\Omega_w = ((1+r)g - 1)V'(c^f). \tag{4.6}$$

From (3.14) I get an expression for the marginal value of real money balance to the employer, Ω_M^f, given by

$$\Omega_M^f = V'(c^f). \tag{4.7}$$

(3.13), (4.6) and (4.7) imply that labor demand is given by

$$f'(e) = (1+r)gw. \tag{4.8}$$

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The employer equates the marginal product of labor to *the effective cost* of labor. Since, the wage bill can be financed only by real money balance carried from the previous period, the effective cost of labor is $(1+r)gw$.

(3.10) and (4.5) imply that consumption of employer, c^f, is given by

$$c^f = f(e) - \xi q = f(e) - c^h. \tag{4.9}$$

The market clearing condition for the goods market, $p_t = \frac{M_t^h}{q_t}$, imply that the desired quantity of goods to sell, q, satisfies

$$q = M^h. \tag{4.10}$$

Using the wage finance constraint (3.11), the employer's budget constraint (3.12), the money market clearing condition, and (4.10), I derive relationship between the desired quantity of goods to sell, q, and the real wage bill, ew,

$$ew = \frac{g\xi + (g-1)\phi(1-\xi)}{g - (g-1)\phi} q. \tag{4.11}$$

For a given real wage bill, ew, (4.11) traces a negative relationship between g and q for any $\phi > 0$. This happens because the real wage bill can be financed in two ways: monetary injection and sales. An increase in g increases the share of real wage bill financed by monetary injection. Thus, the employer has to sell less goods to finance his real wage bill leading to fall in q. When $\phi = 0$, employers do not receive monetary injection and changes in g do not affect q.

Using (4.5), (4.8) and (4.11), I get an alternative expression for the consumption of the worker household, c^h, given by

$$c^h = \xi q = \frac{g - (g-1)\phi}{g\xi + (g-1)\phi(1-\xi)} \xi f'(e)e. \tag{4.12}$$

(4.12) shows that consumption, c^h, and the desired quantity of goods to sell, q, are strictly positive only when, $g\xi + (g-1)\phi(1-\xi) > 0$.

Assumption 2 The parameter values are such that $g\xi + (g-1)\phi(1-\xi) > 0$.

(4.12) is the key equation of the model and captures the distributional consequences of changes in money supply. It shows that for any $\phi > 0$, changes in g affect consumption of worker households *directly*. In particular,

for a given employment, e, an increase in g reduces c^h, for any $\phi > 0$.[9] Thus, a higher g redistributes consumption in favor of employers. This happens because a higher g reduces the desired quantity to sell, q.

The direct effect of monetary policy on c^h is the consequence of the separation between worker households and employers. In a model in which worker households and employers are identical and the representative household receives both wages and dividends, consumption is independent of ϕ and changes in g do not have distributional consequences and consumption is not affected directly by changes in g.[10] Thus, for a given employment level, e, consumption of the household, c^h, is also fixed. As shown below, the distributional effects can significantly change the effects of changes in g on output, unemployment and labor force participation.

Note also that for any given g, changes in ϕ also affect consumption of worker households, c^h, by changing the proportion of real wage bill financed by monetary injection. In particular, if $g > 1$, an increase in ϕ reduces c^h for a given e as a greater proportion of real wage bill is financed by monetary injection. On the other hand, if $g < 1$ (monetary withdrawal), an increase in ϕ increases c^h for a given e as a greater proportion of monetary withdrawal is borne by employers.

By putting (4.8) and (4.12) in (4.3), I get

$$\theta l^{\theta-1} = \frac{\xi}{(1+r)g - 1 + \xi} f'(e) U' \left(\frac{g - (g-1)\phi}{g\xi + (g-1)\phi(1-\xi)} \xi f'(e) e \right). \quad (4.13)$$

(4.13) together with (4.4) and the labor market clearing condition $e = \chi(n)nl$ determine equilibrium e, n, and l and characterize the SSME. Combining (4.4), (4.13), and $e = \chi(n)nl$, the three equations characterizing SSME can be reduced to one equation in one unknown, namely, equilibrium employed labor units, e. For ease of expression, I normalize $\frac{\tau}{a(\theta-1)} = 1$.

Under the normalization that $\frac{\tau}{a(\theta-1)} = 1$, the equilibrium employed labor units, e, satisfies following equation:

$$\theta \left(\frac{e}{a} \right)^{\frac{\zeta(\theta-1)}{\theta(1-\zeta)+\zeta}} = \frac{\xi}{(1+r)g - 1 + \xi} f'(e) U' \left(\frac{g - (g-1)\phi}{g\xi + (g-1)\phi(1-\xi)} \xi f'(e) e \right). \quad (4.14)$$

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[9]Let $X(g) \equiv \frac{g-(g-1)\phi}{g\xi+(g-1)\phi(1-\xi)}$. Then, $X'(g) = -\frac{\phi}{(g\xi+(g-1)\phi(1-\xi))^2} < 0$.

[10]Proof is available on request.

Also equilibrium n and l are given by

$$n = \left(\frac{e}{a}\right)^{\frac{\theta}{\theta(1-\zeta)+\zeta}} \ \& \ l = \left(\frac{e}{a}\right)^{\frac{\zeta}{\theta(1-\zeta)+\zeta}} . \tag{4.15}$$

(4.15) shows that both n and l are increasing functions of e.

Assumption 3. The production function is such that $\frac{df'(e)e}{de} > 0$.

An example of the production function which satisfies the above assumption is $f(e) = e^{\beta}$.

Proposition 1. Under assumptions 1, 2, and 3, there exists a unique SSME characterized by equations (4.1)-(4.13).

For the analysis of the effects of monetary policy, it is useful to illustrate the monetary equilibrium in terms of demand and supply of labor units. (4.8) traces a downward sloping demand (LD) curve in (e, w) space. The supply of labor units is given by:

$$\theta \left(\frac{e}{a}\right)^{\frac{\zeta(\theta-1)}{\theta(1-\zeta)+\zeta}} = \frac{\xi}{(1+r)g - 1 + \xi} wU' \left(\frac{g - (g-1)\phi}{g\xi + (g-1)\phi(1-\xi)} \xi f'(e)e\right). \tag{4.16}$$

It is easy to show that (4.16) traces an upward sloping labor supply (LS) curve in (e, w) space for a given values of g and ϕ. (4.16) also shows that the supply of labor units is directly affected by both parameters of money supply g and ϕ. The existence of equilibrium is illustrated below:

Figure 1
Graphic Portrait of Equilibrium
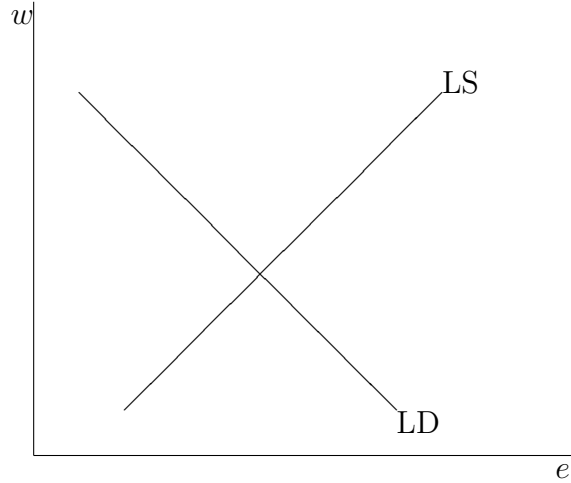
Before analyzing the effects of changes in the money creation rate, g, it is instructive to analyze the effects of changes in the distributional parameter of money supply, ϕ. The following proposition summarizes the effects of changes in distributional parameter, ϕ, for a given money creation rate, g, on output, unemployment, labor force participation, and real wage.

Proposition 2. If the money creation rate $g > 1$, then a higher value of the distributional parameter of money supply, ϕ, increases labor force participation, n, labor intensity, l, employed labor units, e, output, $f(e)$, and unemployment rate, $1 - \chi(n)$, and reduces real wage, w.

The above proposition follows from equation (4.14) and (4.15). The mechanism is as follows. As discussed earlier, a higher value of ϕ for $g > 1$ reduces the worker household consumption, c^h, for a given level of employed labor units, e. The fall in c^h by increasing the marginal return from working induces higher supply of labor units from the worker household. Essentially, changes in ϕ do not affect the demand for labor units curve (equation 4.8). But for $g > 1$, a higher ϕ shifts the supply of labor units curve (equation 4.16) downward to the right in the (e, w) space. Thus, equilibrium employed labor units and output increase and real wage falls. Higher employment increases labor force participation rate and labor intensity. A higher labor force participation rate reduces the entry probability of workers in the labor market leading to higher unemployment rate.

If the money creation rate $g < 1$, then a higher value of the distributional parameter of money supply, ϕ, reduces labor force participation, n, labor intensity, l, employed labor units, e, output, $f(e)$, and unemployment rate, $1 - \chi(n)$, and increases real wage, w. In this case, the supply of labor units curve shifts upward to the left leading to lower equilibrium employed labor units and higher real wage.

Next, I analyze the effects of changes in the money creation rate, g, for a given distributional parameter of money, ϕ, on real activities.

5 Effects of Changes in the Money Creation Rate

(4.16) shows that a higher g affects supply of labor units directly by reducing the marginal value of real money balance and indirectly through consumption of worker households, c^h. I call the first effect *substitution effect of inflation* and the second effect *income effect of inflation* on supply of labor units.

The substitution effect of inflation on supply of labor units is negative, but the income effect is positive. The substitution effect captures the traditional inflation tax effect. For a given worker household consumption, c^h, and real wage, w, a higher money creation rate, g, reduces the return from working inducing the worker household to supply less labor units.

The positive income effect of inflation on supply of labor units is new to the literature and is the consequence of the separation between worker households and employers. In the model, for a given level of real wage, w, and $\phi > 0$, a higher money creation rate, g, reduces worker household consumption, c^h, which by increasing the return from working induces a larger supply of labor units from the worker household.

The above analysis shows that a higher money creation rate may increase or reduce supply of labor units depending on the relative strength of income and substitution effects. The relative strength of these two effects depends on the curvature properties of the worker household's utility function, rate of discount, r, the probability of trading in the goods market, ξ, and the distributional parameter of monetary policy, ϕ.

When $\phi = 0$ (worker households receive all the monetary injection), the income effect is absent and thus a higher money creation rate, g, unambiguously reduces supply of labor units. Since, the demand for labor units curve

remains unaffected, a higher g leads to lower employment and output and higher unemployment rate as in standard monetary models.

To show the effects of changes in the money creation rate, g, when $\phi > 0$, I assume that the worker household has a CRRA preference: $U(c^h) = \frac{c^{h(1-\alpha)}}{1-\alpha}$. Then the following proposition summarizes the effects of higher money creation rate, g, on employment and output.

Proposition 3. Suppose that the worker household has a CRRA preference: $U(c^h) = \frac{c^{h(1-\alpha)}}{1-\alpha}$. If at the initial equilibrium the values of parameters are such that

$$\frac{\alpha\phi[(1+r)g - 1 + \xi]}{[g\xi + (g-1)\phi(1-\xi)][g - (g-1)\phi]} > 1 + r \tag{5.1}$$

then a higher money creation rate, g, increases employed labor units, e, labor force participation, n, labor intensity, l, output, $f(e)$, and unemployment rate, $1 - \chi(n)$, and reduces real wage, w.

The above condition ensures that the positive income effect of inflation on supply of labor units dominates the negative substitution effect. Thus, supply of labor units curve shifts downward to the right in the (e, w) space.

In the case, (5.1) is not satisfied, a higher g reduces employed labor units, e, labor force participation, n, labor intensity, l, output, $f(e)$, and unemployment rate, $1 - \chi(n)$, and increases real wage, w. The effect of changes in g on equilibrium variables is illustrated in the following figure.

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Figure 2
Effects of Increase in g when Condition 5.1 is Satisfied
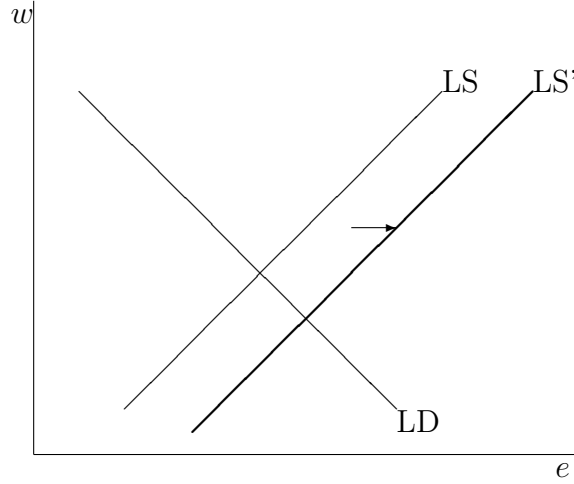
The result that a higher money creation rate, g, can increase output, labor force participation, and unemployment through the redistribution of consumption between workers and employers is new to the literature. In general, since the left hand side of (5.1) is increasing in α, higher the coefficient of relative risk-aversion more likely (5.1) will be satisfied. This happens because higher is α greater is the income effect. In addition, higher is ϕ and r and lower is ξ, more likely this condition will be satisfied.[11]. Reason is that higher is ϕ, larger proportion of wage bill can be financed by monetary injection. On the other hand, higher r and lower ξ increase the cost of carrying real money balance forward. In section 7, I show that this condition is satisfied for the U.S. economy for reasonable values of parameters.

In the next section, I examine the issue of the optimality of market allocations and the distributional consequences of inflation.

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[11]Let $H(\phi,\ \xi) \equiv \frac{\alpha\phi[(1+r)g-1+\xi]}{[g\xi+(g-1)\phi(1-\xi)][g-(g-1)\phi]}$. Then, simple differentiation shows that $\frac{dH}{d\phi} = \frac{\xi g^2+(g-1)^2\phi^2(1-\xi)}{[g\xi+(g-1)\phi(1-\xi)]^2[g-(g-1)\phi]^2} > 0$ and $\frac{dH}{d\xi} = \frac{(1+(g-1)\phi)(\xi-1)}{[g\xi+(g-1)\phi(1-\xi)]^2} < 0$. Note that $(1+(g-1)\phi) > 0$ as for the SSME to exist $g > \frac{1}{1+r}$. Equation 5.1 can be rewritten as $\frac{\alpha\phi[(g-((1-\xi)/(1+r))]}{[g\xi+(g-1)\phi(1-\xi)][g-(g-1)\phi]} > 1$. Then simple differentiation shows that the LHS is increasing in r.

6 Welfare and Distribution

I first begin with characterizing social optimal allocations. The social planner maximizes

$$\max_{c_t^h, n_t, l_t, e_t} \sum_{t=0}^{\infty} \left(\frac{1}{1+r} \right)^t \left[\pi [U(c_t^h) - an_t^{1-\zeta} l_t^{\theta} - \tau n_t] + (1-\pi)V(c_t^f) \right]$$

subject to resource constraints

$$c_t^h + c_t^f = f(e_t) \ \& \tag{6.1}$$

$$e_t = an_t^{1-\zeta} l_t \tag{6.2}$$

where π is the relative weight put by the social planner on the utility of worker households.

In the steady state, it can be shown that the first order conditions reduce to

$$c^h : \pi U'(c^h) = (1-\pi)V'(c^f); \tag{6.3}$$

$$l : \theta l^{\theta-1} = U'(c^h)f'(e); \tag{6.4}$$

$$n : \theta[1-\zeta] = -\zeta + 1 + \frac{\tau}{al^{\theta}n^{-\zeta}}. \tag{6.5}$$

(6.3) characterizes the condition for the socially optimal distribution of consumption. The social planner equates the social marginal utilities of consumption of the worker household and the employer. (6.4) determines the social optimal level of labor intensity. It equates the social marginal cost of supplying one extra unit of labor with its social marginal benefit. (6.5) characterizes the condition for the socially optimal level of labor force participation. While choosing labor force participation, the social planner takes into account the effect of having extra labor market participants on the entry probability of workers in the labor market.

Proposition 4. Under the assumption that $\lim_{c^h \to 0} \pi U'(c^h) = \lim_{c^f \to 0}(1-\pi)V'(c^f)$, there exists a unique allocation, $\{c_s^h, c_s^f, e_s\}$, which satisfies (6.1)-(6.5).

From the comparison of (6.3)-(6.6) with (4.4), (4.12), and (4.13), it is immediately clear that market allocations do not coincide with social optimal allocations. To analyze the optimality properties of market allocations, it is useful to first consider the case of exogenous labor force participation, $\overline{n}$, and thus exogenous matching rate of workers, $\chi(\overline{n})$. In this case, (4.4) and (6.5) do not apply.

Given the identity $e = \chi(\overline{n})\overline{n}l$, the comparison of (6.4) and (4.13) immediately shows that for a given worker household consumption, c^h, the market level of employed labor units, e, approaches its socially optimal level, e_s, as the money creation rate, g, approaches (from above) the Friedman rule, $(1 + r)g = 1$. In other words, for a *given* distribution of consumption, the Friedman rule induces optimal level of employment. However, the comparison of (6.3) with (4.12) immediately shows that the Friedman rule does not lead to the optimal distribution of consumption.

Only when the government has access to other tax and transfer instruments it can induce optimal distribution of consumption. Indeed, if the government can choose the distributional parameter of money supply, ϕ, then together with the Friedman rule it can achieve the socially optimal allocations. In this case, one can easily show that the optimal money creation rate is given by the Friedman Rule, $g_s = \frac{1}{1+r}$, and the optimal distributional parameter of money supply, ϕ_s, is given by

$$\phi_s = \frac{\xi}{r}\left[\frac{f'(e_s)e_s - c_s^h}{\xi f'(e_s)e_s + (1-\xi)c_s^h}\right]. \tag{6.6}$$

The choice of ϕ allows the government to redistribute wealth from agents with relatively low marginal utility of consumption to agents with relatively high marginal utility of consumption.

Now, I consider the case in which labor force participation is endogenous. Suppose that the government can choose ϕ to induce optimal distribution of consumption. Then the comparison of (6.5) and (4.4) shows that the Friedman rule and the optimal choice of ϕ do not induce optimal labor force participation rate and unemployment rate. In fact, with the Friedman rule and the optimal choice of ϕ, the market equilibrium results in too much labor force participation and unemployment relative to the social optimum. The reason is the externality inherent in the search process in the labor market. One additional labor market participant reduces the entry probability of workers in the labor market. Market does not take into account this externality, leading to too many labor market participants.

Now I analyze the effects of monetary policy on consumption and its distribution between worker and employer households. Denote the elasticity of production with respect to employment by $\eta(e) \equiv \frac{f'(e)e}{f(e)}$. Then from (4.12), it follows that the share of the worker household consumption in output, $\frac{c^h}{f(e)}$, is given by

$$\frac{c^h}{f(e)} = \frac{g - (g-1)\phi}{g\xi + (g-1)\phi(1-\xi)}\xi\eta(e). \tag{6.7}$$

The share of employer's consumption in output, $\frac{c^f}{f(e)}$, is given by

$$\frac{c^f}{f(e)} = 1 - \frac{c^h}{f(e)}. \tag{6.8}$$

The following proposition summarizes the distributional consequences of changes in g.

Proposition 5. Suppose that the elasticity of production function with respect to employment, $\eta(e) \equiv \frac{f'(e)e}{f(e)}$, is constant and the value of distributional parameter of money supply, $\phi > 0$. Then,

a. a higher money creation rate, g, reduces the share of worker household consumption in output, $\frac{c^h}{f(e)}$, and increases the share of employer's consumption in output, $\frac{c^f}{f(e)}$;

b. a higher money creation rate, g, reduces worker household consumption, c^h; and

c. if the condition stated in (5.1) is satisfied, a higher g increases employer's consumption, c^f. In addition, a higher g makes employers better-off and worker households worse-off.

An example of the production function which satisfies the condition that the elasticity of production function with respect to employment, $\eta(e) \equiv \frac{f'(e)e}{f(e)}$, is constant is $f(e) = e^\beta$. If the condition stated in (5.1) is not satisfied, a higher g may increase or lower employer's consumption, c^f. Also, it has ambiguous effect on the welfare of employers and worker households.

The proposition shows that a higher inflation rate increases consumption inequality between worker and employer households. Rising output and employment levels entail falling consumption of worker households and rising consumption of employer households. In addition, an increase in g makes

worker households worse-off but employer households better-off. The welfare of worker households falls not only due to fall in consumption, but also due to loss of leisure. Thus, the overall cost of inflation may be low, but the welfare loss of worker households can be substantial. If the social planner puts sufficient weight on the utilities of employers, the social welfare may even increase with inflation.

The reason for the fall in consumption of worker households is that with higher monetary injection employers can finance a larger part of their wage bill through monetary injection. Thus, for a given output they need to sell less in the decentralized goods market, reducing consumption of worker households. Alternatively, one may think of as higher monetary injection increases the price of goods in the decentralized market relatively more, reducing consumption of workers. On the other hand, since employers can consume their own product (or trade in centralized market), higher monetary injection does not negatively affect their profitability (labor demand).

The results that the cost of inflation is primarily borne by households with limited access to financial instruments and higher inflation leads to larger inequality of income/consumption are similar to Erosa and Ventura (2002), Albanesi (2007), Boel and Camera (2009), and Ghossoub and Reed (2017). However as discussed earlier, in these models labor supply is fixed and there is no unemployment.

In the next section, I provide empirical evidence on the long run effects of changes in inflation on output, unemployment rate and labor force participation rate in the United States. I also show that the model generates a positive relationship among inflation rate, output, unemployment, and the labor force participation rate as observed in the U.S. data for reasonable values of parameters.

7 Quantitative Analysis

As discussed earlier, empirical evidence suggests a positive long-run relationship between both inflation and output in the U.S. (Ahmed and Rogers 2000, Ericsson et. al. 2001, and Bashar 2011) and unemployment and inflation (Friedman 1977, Beyer and Farmer 2007, Berentsen, Menzio, and Wright 2011, Haug and King 2014). However, the effect of inflation on labor force participation in the long run has not been examined. In this section, I provide evidence on the long run relationship between labor force partic-

ipation rate and inflation rate in the United States. In addition, I provide further evidence on the long run relationship among output, unemployment, employment and inflation in the United States.

To establish long-run relationship between inflation and other variables, I use co-integration technique. To test for co-integration between inflation, g_t, and other variables, I estimate following relationship:

$$y_t = b_1 + b_2 g_t + B_3 X_t + \gamma_t \tag{7.1}$$

where y_t is the dependent variable, X_t is the matrix of other deterministic variables such as time trend and intercept dummies indicating period of recession and recovery, B_3 is the associated vector of coefficients and γ_t is the error term. These dummies have been included because it has been argued that rejection of long-run neutrality or super-neutrality for the U.S. may be due to not accounting for recessions (Boschen and Ortok 1994).

According to Engel-Granger (1987) representation theorem if both y_t and g_t are integrated of order 1, $I(1)$, but the residual, γ_t, is integrated of order zero, $I(0)$, then two variables are said to be cointegrated. Cointegration can also be established through the error-correction modeling technique outlined below:

$$\Delta y_t = \sum_{i=1}^{n_1} h_i \Delta y_{t-i} + \sum_{i=0}^{n_2} k_i \Delta g_{t-i} + d\gamma_{t-1} + \Gamma_t \tag{7.2}$$

where Γ_t is the error term.

Equation 7.2 incorporates short-run dynamics into the adjustment process. When y_t and g_t are adjusting towards their long-run equilibrium, the gap between these two variables decreases. Since the gap between the two variables is measured by γ_t, cointegration is established if the coefficient of γ_{t-1}, d, is negative and significant. Kremers *et. al.* (1992) have shown that this approach towards cointegration is a more efficient method. To establish cointegration, I use both approaches.

For estimation, I use seasonally adjusted quarterly data of labor force participation rate (LFPR), n_t, private sector real GDP (base year 2009), $f(e_t)$, unemployment rate (UR), $1 - \chi(n_t)$, employment, e_t, and inflation rate, g_t, taken from *FRED Database, St. Louis Federal Reserve Bank.* Inflation rate is measured by the growth rate in GDP deflator and employment by number of full-time employed workers. All data except for employment are from the first quarter of 1948:1 to the third quarter of 2017:3, the latest

period for which data available. Employment data is from the first quarter of 1968:1. The labor force data is for civilian population 16 years and above. The data for the period of recessions and recoveries are from the NBER (Business Cycle Reference Dates, the Business Cycle Dating Committee). The details of data sources are given in the Appendix.

Figure 1 depicts the LFPR for the period 1948Q1-2017Q3. It shows that the LFPR increased almost continuously till the end of 1990's and afterwards it started declining. The declining trend in the LFPR post-2000 is attributed to structural changes such as retirement of baby-boomers and labor-force participation for female reaching its peak in 1990's (Juhn and Potter 2006, Aaronson et. al. 2006). To examine the long-run relationship between LFPR and inflation, I divide data in two-time periods 1948:1-1999:4 and 2000:1-2017:3. For consistency, I also divide other data series in two-time periods.

First, I perform unit root test based on Augmented Dicky-Fuller (ADF) and the KPSS tests for level and trend stationarity of inflation, LFPR, unemployment rate, and the log of private sector real GDP and employment. I perform unit root tests for the entire period (1948:1-2017:3) and also both sub-periods separately. Since the LFPR shows structural break in 2000s, I also use Perron test for unit root, since this test allows for structural break. All these tests indicate that all five variables are integrated of order one, $I(1)$.[12]

Table 1 shows estimated models based on 7.1. The first panel shows results for the first sub-period and the second panel shows results for the second sub-period. The first panel shows that estimated coefficients of inflation rate are significant and positive at 1% or 5% level of significance in all specifications for the first sub-period 1948:1-1999:4. The ADF test (with no constant and trend) on the residuals of the estimated equation rejects the null of unit root at 1% or 5% in all the regressions. These results suggest that inflation rate is cointegrated with private sector real GDP, employment, unemployment rate, and labor force participation rate. In addition, inflation rate has a positive and significant effect on these variables in the first sub-period.

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[12]Note that by design LFPR and unemployment rate are bounded between 0 and 1. However, as argued by King and Watson (1992) in a small sample highly persistent processes are better modelled as integrated processes.

Table 1: Bivariate Cointegration

Variables	$LFPR_t$	$f(e_t)$	UR_t	e_t
	Sub-Period I	1948:1-1999:4		
g_t	0.1083*	0.0104*	0.0716**	0.0038**
	(0.0313)	(0.0017)	(0.0361)	(0.0015)
ADF Statistics	-3.005^{**}	-6.56^{*}	-4.24^{*}	-2.90^{**}
R^2	0.96	0.99	0.33	0.99
No. of Obs	208	208	208	128
	Sub-Period II	2000:1-2017:3		
g_t	0.0974**	0.0156*	-0.4714^{**}	0.0122**
	(0.0432)	(0.0033)	(0.2221)	(0.0033)
ADF Statistics	-3.72^{*}	-3.32^{**}	-1.16	-1.91
R^2	0.96	0.95	0.19	0.51
No. of Obs	70	70	70	70

Note:
1. * and ** indicate significance levels of 1% and 5% respectively. Errors are in parenthesis.
2. The estimated models include trend and intercept dummies for recessions.
3. The first sub-period for employment is 1968:1-1999:4.

The second panel shows results for the second sub-period. It shows that inflation is positively and significantly associated with private-sector real GDP, LFPR and employment. However, unit root test shows that inflation is cointegrated with output and LFPR, but not with employment (null of unit root in the residual cannot be rejected). The result also shows that inflation is significantly and negatively associated with unemployment, unlike the first sub-period. But in this case as well unit root test shows that inflation and unemployment are not cointegrated. The lack of cointegration between inflation rate, unemployment rate, and employment in the second sub-period may be in part due to smaller sample. In addition, this sub-period had two unusual recessions: 2001-02, which was marked by very slow recovery in employment (Jobless Recovery) and 2007-09: the Great Recession.[13]

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[13]Estimation for the entire period 1948:1-2017:3 for unemployment and 1968:1-2017:3 for employment shows that inflation rate is positively and significantly associated with both unemployment and employment. Also unit root test suggests that inflation is cointegrated with both unemployment rate and employment.

To further, confirm the existence of cointegration, I estimate the error-correction model outlined in (7.2) for private-sector real GDP, unemployment, employment and labor force participation. The estimated models are given in table 2 below.

Table 2: Error-Correction Models

Variables	$\Delta LFPR_t$	$\Delta f(e_t)$	ΔUR_t	Δe_t
	Sub-Period I	1948:1-1999:4		
γ_{t-1}	-0.0358^{**}	-0.1243^{*}	-0.0738^{*}	-0.1151^{*}
	(0.0161)	(0.0407)	(0.0164)	(0.0358)
R^2	0.07	0.27	0.51	0.40
DW	2.02	1.98	1.98	1.96
No. of Obs	201	203	203	123
	Sub-Period II	2000:1-2017:3		
γ_{t-1}	-0.1338^{**}	-0.1104^{**}	-0.0196	-0.0249
	(0.0675)	(0.0561)	(0.0103)	(0.0243)
R^2	0.16	0.33	0.75	0.49
DW	2.05	1.85	2.12	1.94
No. of Obs	70	69	70	70

Note:
1. * and ** indicate significance levels of 1% and 5% respectively. Errors are in parenthesis.
2. The estimated models include lagged values of dependent variables and difference in inflation rate. Lag lengths were chosen on the basis of minimizing Akaike and Schwartz information criteria.
3. The first sub-period for employment is 1968:1-1999:4.

The estimated models confirm previous results. In the first sub-period inflation rate is cointegrated with LFPR, unemployment rate, private sector real GDP, and employment. In the second sub-period, it is cointegrated with LFPR and private real GDP, but not with unemployment rate and employment. Overall, these results suggest that inflation rate has a positively effect on output, employment, unemployment, and labor force participation in the United States particularly in the last century.

Next, I show that the model generates a positive relationship among inflation, output, employment, unemployment, and labor force participation for reasonable values of parameters. Assume that both worker households and employers have CRRA utility function:

$$U(c) = \frac{c^{1-\alpha}}{1-\alpha}. \tag{7.3}$$

As shown in (5.1), the effects of changes in inflation rate depends on the values of r, g, ξ, α and ϕ.

To begin with I set time period to be a year as in Lucas (2000) and Rocheateau and Wright (2008). Then, I set the rate of discount, $r = 0.03$ as in Lucas (2000) and Rocheteau and Wright (2008) and $\xi = 0.5$ as in Rocheteau and Wright (2008). To set the value of g, I estimate the annual rate of inflation in the U.S. for the period 1948:1-2017:3. The estimated value is 3.5%. I set $g = 1.035$ to match the observed average inflation rate.

Now, I consider two values of $\phi = 0.5$ & 1. When $\phi = 0.5$, both worker households and employers receive same amount of monetary injection as in standard monetary models. When $\phi = 1$, firms receive all the monetary injection as in limited participation models (e.g. Fuerst 1992, Christiano, Eichenbaum, and Evans 1997).

Suppose now that $\phi = 0.5$, then (5.1) is satisfied for any $\alpha > 1.92$. In the macro literature, the value of $\alpha = 2$ is commonly used (e.g. Alvarez, Atkeson, and Kehoe 2002, Erosa and Ventura 2002, Heer 2003).[14] If $\phi = 1$ then (5.1) is satisfied for any $\alpha > 0.97$. In general, lower is ϕ, higher is α required to generate positive relationship between inflation rate, output, employment, unemployment, and labor force participation. For example, if $\phi = 0.4$, then α should be greater than 2.45.

Now I vary ξ and consider a very high value of $\xi = 0.99$. This implies that ninety-nine percent of buyers and sellers in the goods market are able to do their desired trading. This changes results only marginally. In the case, when $\phi = 0.5$, the model generates positive relationship between inflation rate, output, unemployment, and labor force participation for any $\alpha > 1.99$. For $\phi = 1$, the required $\alpha > 1$.

Finally, I consider the case in which time period is equal to a quarter rather than a year. In this case, I set $r = 0.008$ and $g = 1.009$. Change in time period does not alter the results much. For example, with $\xi = 0.5$ and $\phi = 0.5$, the required $\alpha > 1.97$ and for $\phi = 1$, the required $\alpha > 0.99$. Similar is the case when $\xi = 0.99$. The above discussion shows that the model

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[14]Empirical estimates of α varies quite considerably. For example, Kydland and Prescott (1982) use value of $\alpha = 1.5$ and Cooley and Hansen (1989) use logarithmic preferences ($\alpha = 1$). Hodrick et. al. (1991) use values in the range of 0-9.5, and Wang and Shi (2006) use value of 4.

generates a positive relationship among inflation rate, output, employment, unemployment, and labor force participation for reasonable values of parameters.

8 Conclusion

In this paper, I studied the effects of changes in the inflation rate in a model with endogenous labor force participation and separation between workers and employers (owners) of the firms. In the model, a higher inflation rate increases output, employment, labor force participation, and unemployment. These results are consistent with the empirical evidence which shows that higher inflation is associated with higher output, labor force participation, employment and unemployment in the United States.

In the model, a higher inflation rate increases consumption inequality between workers and employers. Inflation acts as a regressive consumption tax, and the cost of inflation is borne by workers. Increase in inflation makes worker households worse-off but employer households better-off. These results are consistent with substantial empirical evidence that cost of inflation is mainly borne by poorer households, with limited access to financial system and inflation and income inequality are positively related.

The market equilibrium is inefficient. The Friedman rule for a given distribution of consumption and labor force participation leads to efficient employment. However, the Friedman rule does not ensure efficient distribution of consumption and labor force participation.

In the model, I assumed that workers can only buy goods using money. One interesting extension can be to allow workers to buy goods using both money and other means of payment (e.g. credit) similar to Erosa and Ventura (2002), Albanesi (2007), and Boel and Corbae (2009). This is likely to reduce the welfare cost of inflation for workers and the response of labor supply to inflation. However, these studies show that with realistic values of parameters, the cost of inflation mainly falls on poorer households.

In the model, I also assumed that number of buyers and sellers are fixed and worker and only a fixed fraction of buyers and sellers are able to enter the goods market in any time period. One can endogenize number of buyers or their search-intensity (as in Shi 1998, Liang *et. al.* 2007) as well as number of sellers (as in Rocheteau and Wright 2005, 2008). These extensions will also allow me to endogenize the entry rates for both buyers and sellers in the

goods market. The endogenity of number of buyers or their search-intensity is likely to magnify the positive effects of inflation on output and employment as in Shi (1998) and Liang *et. al.* (2007). On the other hand, the endogenity of number of sellers is likely to weaken the positive effects of inflation on output and employment as in Rocheteau and Wright (2005).

Appendix 1: Proofs

Proposition 1. The equilibrium employment, e, is given by

$$\theta \left(\frac{e}{a}\right)^{\frac{\zeta(\theta-1)}{\theta(1-\zeta)+\zeta}} = \frac{\xi}{(1+r)g - 1 + \xi} f'(e) U' \left(\frac{g - (g-1)\phi}{g\xi + (g-1)\phi(1-\xi)} \xi f'(e)e\right).A1$$

The LHS of A1 is increasing in e. Simple differentiation of the RHS of A1 with respect to e shows that it is decreasing in e if $\frac{f'(e)e}{de} \geq 0$. In addition, given the assumptions that $\lim_{c^h \to 0} U'(c^h) > 0$, $\lim_{e \to 0} f'(e) \to \infty$, and $\lim_{e \to 0} f'(e)e < \infty$

$$\lim_{e \to 0} \text{LHS} = 0 < \lim_{e \to 0} \text{RHS}.$$

Under above conditions, if we plot the LHS and the RHS of A1 against e, then we get a unique intersection of the LHS and the RHS at some $0 < e < \infty$. Thus there exists a unique and finite e which solves A1.

Proposition 2.

$$c^h \equiv \frac{g - (g-1)\phi}{g\xi + (g-1)\phi(1-\xi)} \xi f'(e)e. \qquad\qquad \text{A2}$$

For a given e,

$$\frac{dc^h}{d\phi} = \frac{g(g-1)}{(g\xi + (g-1)\phi(1-\xi))^2} \xi f'(e)e. \qquad\qquad \text{A3}$$

Thus for a given e, $\frac{dc^h}{d\phi} < 0$ if $g > 1$ and $\frac{dc^h}{d\phi} > 0$ if $g < 1$. Thus for a given e, a higher ϕ shifts the supply of labor unit curve downward to the right in (e, w) space if $g > 1$. Opposite happens when $g < 1$.

Proposition 3. With CRRA utility function A1 becomes

$$\theta e^{\frac{\zeta(\theta-1)}{\zeta+\theta(\zeta-1)}} = \frac{\xi^{1-\alpha}}{(1+r)g - 1 + \xi} \frac{f'(e)}{(f'(e)e)^\alpha} \left[\frac{g\xi + (g-1)\phi(1-\xi)}{(g - (g-1)\phi)}\right]^\alpha. \qquad \text{A4}$$

The effect of the money creation rate, g, on equilibrium employment depends on how it affects $T(g) \equiv \frac{1}{(1+r)g-1+\xi} \left[\frac{g\xi+(g-1)\phi(1-\xi)}{(g-(g-1)\phi)}\right]^\alpha$. Simple differentiation of $T(g)$ with respect to g shows that $\frac{dT(g)}{dg} > 0$ if

$$\frac{\alpha\phi[(1+r)g - 1 + \xi]}{[g\xi + (g-1)\phi(1-\xi)][g - (g-1)\phi]} > 1 + r. \qquad \text{A5}$$

In the case, condition A5 is satisfied a higher g shifts up the RHS to the right for a given e. Thus equilibrium employment, e rises. An increase in e increases n, l, $f(e)$, and $1 - \chi(n)$ and reduces wage, w.

Proposition 4.

The social planner allocations are characterized by following two equations:

$$\pi U'(c^h) = (1 - \pi)V'(c^f) \equiv (1 - \pi)V'(f(e) - c^h) \ \& \qquad \text{A6}$$

$$\theta(1 - \zeta)^{\frac{(c-1)(\theta-1)}{\zeta+\theta(1-\zeta)}} e^{\frac{\zeta(\theta-1)}{\zeta+\theta(1-\zeta)}} = U'(c^h)f'(e). \qquad \text{A7}$$

A6 gives a relationship between c^h and e. The total differentiation of A6 shows

$$\frac{dc^h}{de} > 0. \qquad \text{A8}$$

Since, $f(0) = 0$ and $c^f \geq 0$, under the assumption that $\lim_{c^h \to 0} U'(c^h) = \lim_{c^f \to 0} V'(c^f)$, A6 implies that $c^h = 0$, when $e = 0$.

A7 gives another relationship between c^h and e. The total differentiation of A7 shows

$$\frac{dc^h}{de} < 0. \qquad \text{A9}$$

Given the assumptions that $\lim_{e \to 0} f'(e) = \infty$, and $\lim_{c^h \to 0} U'(c^h) > 0$, A7 implies that $c^h \to \infty$ when $e = 0$. The above discussion implies that the curves traced by A6 and A7 intersect only once in the (e, c^h) space.

Proposition 5.

Part a. Part a. follows from simple differentiation of (6.7) with respect to g which shows that

$$\frac{d(c^h/f(e))}{dg} = -\frac{\phi}{(g\xi + (g-1)\phi(1-\xi))^2}\xi\eta(e) < 0. \qquad \text{A10}$$

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Thus the share of consumption of worker's household in GDP is decreasing in g and that of employers is increasing in g.

Part b. Case I: Equilibrium output is declining in g. In this case, part b. of the proposition follows from part a.

Case II: Equilibrium output is increasing in g.

The equilibrium employment is given by

$$\theta \left(\frac{e}{a}\right)^{\frac{\zeta(\theta-1)}{\theta(1-\zeta)+\zeta}} = \frac{\xi f'(e)}{(1+r)g - 1 + \xi} U'\left(c^h\right). \qquad \text{A11}$$

Note that LHS of A11 is increasing in e. The first term in the RHS is decreasing in both g and e. This term falls with an increase in g. Thus, for a new equilibrium to be achieved, it must be the case that the second term in RHS, $U'(c^h)$ rises, which requires c^h to fall.

Part c. (6.8) and part a. imply that a higher g increases c^f and the welfare of employers, when (5.1) is satisfied. Also, higher l, n, and lower c^f imply that a higher g reduces the welfare of worker households.

Data Source

1. Real Gross Domestic Product, Billions of Chained 2009 Dollars, Quarterly, Seasonally Adjusted Annual Rate (Series GDPC1).

2. Real Government Consumption Expenditures and Gross Investment, Billions of Chained 2009 Dollars, Quarterly, Seasonally Adjusted Annual Rate (Series GCEC1).

3. Private-Sector Real GDP = GDPC1 - GCEC1.

4. Gross Domestic Product: Implicit Price Deflator, Percent Change from Preceding Period, Quarterly, Seasonally Adjusted Annual Rate (Series A191RI1Q225SBEA).

5. Monthly Seasonally Adjusted Civilian Labor Force Participation Rate: 16 Years and Above (Bureau of Labor Statistics: Series LNS11300000)

6. Monthly Seasonally Adjusted Civilian Unemployment Rate: 16 Years and Above (Bureau of Labor Statistics: LNS14000000)

7. Monthly Seasonally Adjusted Employed, Usually Work Full Time, Thousands of Persons (Bureau of Labor Statistics: LNS12500000)

8. Business Cycle Reference Dates (Source: NBER available
   at *http://www.nber.org/cycles.html*)

Source: FRED Database, St. Louis Federal Reserve Bank, available at *https://fred.stlouisfed.org*.

Note: Monthly series were converted into quarterly series.

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